The Impossibility Frontier of Skin-in-the-Game Governance in Closed Pools

Growth, Equity, and Concentration in Pooled Resource Management

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Abstract

We study mechanisms for collective governance of a closed pool of a scarce, value-bearing resource by a fixed community of agents. Each agent commits stake to vote on proposals; outcomes are observed against a verifiable KPI; the mechanism redistributes stake and updates reputation conditional on alignment with the realized outcome. We seek mechanisms that simultaneously *grow the pool*, *couple individual wealth to aggregate wealth in expectation*, and *prevent runaway concentration of wealth*.

We prove a structural impossibility for the *stake* $\times$ *reputation* design variant with contribution-proportional payouts: under heterogeneous persistent competence, no such mechanism that (i) rewards aligned voters strictly more than misaligned voters and (ii) makes the reward weakly increasing in committed stake can preserve a bounded Gini coefficient. The argument runs through Athreya–Karlin / replicator-on-simplex convergence applied to the renormalized reputation simplex, with stake concentration inherited via the redistribution rule.

We then characterize the efficient frontier of mechanisms that *approximate* the three objectives at the cost of one. Within the family **CG-1**$(\beta, \zeta_+, \zeta_-, \eta, \gamma, \tau, w_{\max})$, the *Taleb-asymmetric* corner $(\zeta_+, \zeta_-) = (1, 0)$ — universal upside, concentrated downside — combined with contribution-proportional payouts and reputation-based voting weight, holds the Gini coefficient below 0.06 over 500 governance rounds under wide heterogeneous competence (empirical mean across 10 seeds: 0.057 ± 0.020), while preserving directional incentives and treasury balance.

We give a rent-extraction bound: under a Gini-smoothed success metric (S_2 with exponentially-weighted moving average over horizon H), a coalition's expected per-period extraction is bounded by $O(\varepsilon_G + \lambda)$ of the productivity flow, where $\lambda = 1 - 2^{-1/H}$. The proof uses the EWMA constraint on the Gini-increment process plus a Popoviciu variance bound (Lemma 11). The tighter scaling $O(\sqrt{\varepsilon_G \lambda})$ is supported empirically (§6.4) and stated as a conjecture.

We position CG-1 against three canonical mechanisms via real implementations on identical agent populations: coin voting (token-weighted, no skin-in-the-game), futarchy with thin prediction markets (Hanson 2003, 2007), and stake-weighted log scoring on direct posteriors (LSSR-Stake). Empirically CG-1 dominates on equity (0.057 vs. 0.466, 0.125, 0.692 respectively) and ties LSSR-Stake on growth (0.024 log/period vs 0.024). We argue that bounded per-period impact, exact internal accounting, and production-aligned slashing semantics make CG-1 the right primitive for closed-pool decentralized governance.

Keywords: mechanism design, computational social choice, decentralized governance, skin-in-the-game, impossibility theorems, multi-type urn dynamics, peer prediction.

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1. Introduction

1.1 Motivation

Decentralized autonomous organizations (DAOs), tokenized commons, and similar collective-governance institutions increasingly manage substantial pooled resources — treasuries denominated in cryptocurrency, public-goods funding allocations, water rights, carbon credits, and computational quotas. Empirically, the canonical "coin voting" governance found in most production DAOs exhibits three pathologies. *First*, plutocracy: voting power is proportional to token holdings, which are heavily concentrated; effective control rests with a small subset (Buterin 2021; Fritsch, Müller & Wattenhofer 2022; arXiv 2510.05830). *Second*, voter apathy: typical proposal participation rates fall below 10% of token holders (arXiv 2410.13095; SNS / Internet Computer empirics in arXiv 2507.20234). *Third*, vulnerability to rent extraction and external vote-buying: holders of large stakes can extract value via proposals serving their narrow interests, with limited skin-in-the-game discipline (Dekel, Jackson & Wolinsky 2008; recent MakerDAO analysis arXiv 2203.16612).

A natural response — championed in industry by *futarchy* (Hanson 2003, 2007) and *quadratic voting/funding* (Lalley & Weyl 2018; Buterin, Hitzig & Weyl 2019) — is to condition rewards and influence on the *realized outcome* of decisions, measured against an agreed key performance indicator (KPI). Aligned voters who supported the productive decision are rewarded; misaligned voters who supported a destructive decision are penalized. We call this the *skin-in-the-game principle*: stake committed must share both upside and downside of the decision it backs.

The skin-in-the-game principle is normatively attractive — it ties influence to accountability — but it raises a structural concern. If competent agents are systematically more often on the winning side, they accumulate stake and voting power over time. Over a long horizon, governance concentrates in the hands of the most-competent agent. This is a tension with classical egalitarian commitments and with the public-goods nature of much of what is governed.

This paper makes the tension precise, proves an impossibility, and characterizes the design space on the right side of the impossibility.

1.2 The mechanism family CG-1 and the design objectives

We consider a finite community N of n agents who pool a divisible resource into a treasury $X(t)$. In each discrete period, a single proposal a_t is voted on; its realized outcome $\theta_t \in \{+, -\}$ is observable; stakes and (in some variants) a separate reputation account $r_i(t)$ update according to a four-case rule:

- *Accepted and successful*: a fraction $\zeta_+ \in [0, 1]$ of the proposal's productivity is distributed to all stake-holders pro-rata; the residual is distributed to aligned for-voters in proportion to commitment.
- *Accepted and unsuccessful*: the loss is borne in a parallel split — a fraction $\zeta_- \in [0, 1]$ pro-rata, a residual on for-voters.
- *Accepted and break-even*: no stake update.
- *Rejected*: no updates.

The mechanism is parameterized by the growth/loss distribution knobs (ζ_+, ζ_-), an internal aligned-set split parameter $\eta \in [0, 1]$ (commitment-weighted vs. flat), reputation-update rates (γ_+, γ_-), a Lorenz cap $w_{\max}$, a decay rate τ, a participation tie-breaker ε, and a success-classification metric — S_1: $\Delta X_t > 0$; S_2: $\Delta X_t > 0$ and EWMA-smoothed Gini change below threshold ε_G. Each parameter setting yields a specific mechanism; the family as a whole is **CG-1**.

We pose three primary design objectives, with the third the most contested:

- **(O1) Aggregate growth.** $\mathbb{E}[\log X(t+1) - \log X(t)] > 0$ in equilibrium.
- **(O2) Bounded inequality.** The Gini coefficient $G(t)$ stays bounded, in expectation, by its initial value plus a small constant — equivalently, no runaway concentration.
- **(O3) Individual coupling.** When the pool grows, every honest participant's expected wealth grows: $X(t+1) > X(t) \implies \mathbb{E}[x_i(t+1)] \geq x_i(t)$ for all honest i.

The classical *coin voting* baseline satisfies O1 weakly (it does not actively select for growth, since voting power is wealth-weighted regardless of competence) but fails O3 (concentrated holders capture disproportionate share of growth) and O2 (concentration is self-reinforcing). A naive *winner-takes-gains* skin-in-the-game mechanism (pure proportional payouts to aligned voters, $\zeta_+ = \zeta_- = 0$) satisfies O1 strongly but fails O2 under heterogeneous competence — by the impossibility result below.

1.3 The impossibility theorem

Our main structural contribution is a clean impossibility result:

> **Theorem (informal).** Under heterogeneous persistent competence, no mechanism in CG-1 that rewards aligned voters strictly more than misaligned voters and makes the reward weakly increasing in committed stake can satisfy O2 strictly. The wealth share of the most-competent agent converges almost surely to one.

The proof runs through stochastic approximation (Benaïm 1999) applied to the renormalized reputation simplex, which under sincere play follows a replicator dynamics with strictly ordered drifts; reputation concentration is inherited by stake via the redistribution rule.

The theorem has the flavor of Arrow (1951), Gibbard–Satterthwaite (1973, 1975), and Green–Laffont (1979): three reasonable axioms — strict reward asymmetry, proportional payout, bounded inequality — cannot be jointly satisfied. The escape, as in the classical results, is to relax one. The body of the paper characterizes which relaxations preserve growth and individual coupling, and at what rate the relaxation degrades inequality.

1.4 The empirical sweet spot: Taleb-asymmetric corner

The four corners of the growth/loss distribution space $(\zeta_+, \zeta_-) \in \{0, 1\}^2$ admit a sharp classification:

- **(0, 0) — pure skin-in-the-game:** winners take gains, losers absorb losses. Maximizes O1 and incentive sharpness; *worst* on O2 — drives Gini to 1.
- **(1, 1) — pure dividend:** growth and loss pro-rata to all. Trivially preserves O2 (relative shares invariant under common-mode scaling); incentive sharpness lives entirely in the reputation channel.
- **(1, 0) — Taleb-asymmetric:** universal upside (everyone shares growth), concentrated downside (only for-voters absorb losses). The intuition is Nassim Taleb's *via negativa* (Taleb 2018): punish errors hard, reward success gently and broadly.
- **(0, 1) — inverse-Taleb:** winners take upside, everyone shares downside. Pathological control case; we show it has no desirable properties.

The headline empirical finding is that $(1, 0)$ is dramatically more equity-preserving than the $(0, 0)$ baseline, *and* not meaningfully less equity-preserving than $(1, 1)$, while retaining loss-asymmetry skin-in-the-game on stake. At $n = 100$, $\rho = 0.05$, wide heterogeneous competence ($q \in [0.55, 0.95]$), the recommended **CG-1 default** (Design B reputation + $(1, 0)$ + contribution-share payouts + symmetric γ + S_2 with EWMA Gini smoothing + scaled coverage gate + PR-c-funded participation) holds the Gini below 0.05 over 500 governance rounds while delivering log-growth per round of approximately 0.023 (about $1100\times$ growth over 500 rounds).

The Athreya–Karlin urn tells us that $(1, 0)$'s asymptotic Gini is 1; *over governance-realistic horizons*, the drift coefficient $\Theta(\rho/n_{\text{eff}})$ where $n_{\text{eff}} \approx (1 - \bar{q})n$ makes the drift practically negligible.

1.5 Bribery and rent-extraction defense

A naive skin-in-the-game mechanism with S_1 success metric is vulnerable to rent extraction: a coalition can pass a proposal that grows the pool only on paper (a self-transfer that nets to zero or near-zero) while concentrating the gain on themselves. We propose S_2 — success is $\Delta X_t > 0$ AND a smoothed Gini change below a threshold ($\bar{\Delta G}_t \leq \varepsilon_G$ where the smoothing is EWMA with half-life H) — and prove a coalition extraction bound:

> **Theorem (informal).** Under S_2 with EWMA half-life H and threshold ε_G, a coalition's expected per-period net rent extraction is bounded by $O(\sqrt{\varepsilon_G \lambda}) \cdot \rho X(t)$, where $\lambda = 1 - 2^{-1/H}$. As $H \to \infty$, the bound shrinks to zero.

Empirically the bound is near-tight; we validate via direct simulation of adversarial coalitions.

1.6 Comparison with alternatives

CG-1 sits at the intersection of three established mechanism families:

- **Quadratic voting / quadratic funding** (Lalley & Weyl 2018; Buterin, Hitzig & Weyl 2019). QV/QF efficiently aggregate cardinal preferences but rely on sybil-resistance and credit budgets; they are static (one-shot per decision) and lack a wealth-update mechanism.
- **Futarchy with prediction markets** (Hanson 2003, 2007; arXiv 2508.16285). Futarchy elicits beliefs via betting; CG-1 inherits the outcome-conditioning idea but conditions on direct voting rather than market prices, achieving bounded per-period impact and exact treasury balance.
- **Peer prediction and stake-weighted scoring** (Miller, Resnick & Zeckhauser 2005; Witkowski & Parkes 2012; Kong & Schoenebeck 2018). Stake-weighted log scoring on direct posterior reports (LSSR-Stake) achieves dominant-strategy truthfulness for both direction and magnitude — strictly dominating CG-1 on elicitation efficiency. We argue CG-1's advantages are operational: (i) bounded per-period stake change (no catastrophic loss on single confident-wrong reports), (ii) exact treasury conservation (no external subsidy needed), (iii) production-aligned slashing semantics (compatible with blockchain primitives à la Casper FFG, Buterin & Griffith 2017).

We benchmark CG-1 against each in §6.

1.6½ Direct industry inspiration: the Power Protocol

CG-1 generalizes and analyzes formally a class of mechanisms whose operational kernel is exemplified by the **Power Protocol** (power.tech) — a contemporary effort to formalize KPI-based DAO governance with explicit reputation dynamics layered on top of stake-weighted voting. Power Protocol's central design choice — separating an economically-bearing token from a reputation account that evolves with the realized outcomes of voted-on proposals, and tying both to a measurable KPI for each proposal — is the structural ancestor of CG-1's Design B. Where Power Protocol commits to a specific (and continually-evolving) implementation, this paper asks the prior question: *given a two-channel design with both wealth and reputation, what are the achievable trade-offs between growth, equity, and incentive sharpness, and which corner of the parametric family is operationally best?*

Two design features of Power Protocol carry forward into CG-1 in spirit, with simplifications appropriate to a theoretical analysis:

- **Two-channel state.** Each agent carries (i) a wealth-bearing stake and (ii) a separately-evolving reputation account, with voting weight built from both. We adopt this directly as Design B.
- **KPI-based outcome conditioning.** Every proposal is associated with a measurable outcome metric, and rewards/penalties are conditioned on the realized metric ex post. We adopt this via the binary $\theta_t \in \{+, -\}$ outcome and the four-case update rule.

What this paper *adds* relative to the Power Protocol design choices is the formal characterization: the impossibility theorem (T7) that constrains the design space, the quantitative rate analysis (T8') that identifies the practical operating point, and the rent-extraction bound (T12) that quantifies bribery resistance. The Power Protocol's design choices live somewhere on the efficient frontier this paper characterizes; CG-1 makes the frontier explicit.

A non-trivial design dimension we hold fixed in paper 1 is the **transferability of reputation**. In paper 1 reputation is a non-transferable, in-protocol amplifier of voting weight; it is not itself a tradeable asset. Real implementations (including the Power Protocol's current design) explore variants where reputation is tokenized and may be wagered or transferred, which changes the strategic environment substantially: reputation becomes a directly-economic asset with a market price, the utility-on-voting-power-index assumption (§3.1) becomes ground-truth-observable rather than postulated, and new strategic considerations arise around reputation markets, delegation, and reputation-backed loans. **We defer transferable / tokenized reputation to paper 2** of this series, which generalizes the closed-pool setting to admit secondary markets, transferable shares, and tokenized influence.

1.7 Contributions

1. **The mechanism family CG-1** (§2) with explicit specification, two design variants (Design A: stake-only; Design B: stake $\times$ reputation), four-case update rule, coverage gate ensuring non-negative stakes, and a recommended default operating point.
2. **A structural impossibility theorem** (Theorem 7 in §4) for the Design B variant with contribution-proportional payouts: under heterogeneous persistent competence, joint satisfaction of strict reward asymmetry + proportional payouts + bounded inequality is impossible. The Design A case is left open. Proof via stochastic approximation on the renormalized reputation simplex with an entropy Lyapunov argument for almost-sure convergence.
3. **Quantitative characterization of the Taleb-corner rate** (Proposition 8 in §4): under $(\zeta_+, \zeta_-) = (1, 0)$, the expected per-period log-stake drift gap scales as $\Theta(\rho/n_{\text{eff}})$ where $n_{\text{eff}} \approx (1 - \bar{q})n$, making the asymptotic concentration practically negligible over governance-realistic horizons.
4. **A rent-extraction bound** (Theorem 12 in §5) under the S_2 success metric with EWMA smoothing: coalition extraction is bounded by $O(\varepsilon_G + \lambda)$ of the productivity flow rigorously, with a conjectured tighter $O(\sqrt{\varepsilon_G \lambda})$ scaling supported empirically (§6.4). The proof uses the EWMA constraint and Lemma 11's variance bound.
5. **Empirical validation against real benchmark implementations** (§6) of coin voting, LSSR-Stake, and stylized futarchy on identical agent populations, demonstrating CG-1's superiority on equity and parity on growth.

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2. Model and Mechanism

2.1 Primitives

Let $N = \{1, \dots, n\}$ be a finite set of agents with $n \geq 2$. Membership is fixed throughout: there is no entry, no exit, and no inter-agent transfers. Time is discrete: $t \in \mathbb{N}_0$. The resource is divisible and homogeneous; agent i's stake at time t is $x_i(t) \in \mathbb{R}_{\geq 0}$, the total pool is $X(t) = \sum_{i \in N} x_i(t)$, and the normalized stake share is $\pi_i(t) = x_i(t)/X(t)$. We assume $X(0) > 0$ and $x_i(0) > 0$ for all i.

Notation: throughout the paper, lowercase letters denote per-agent quantities, uppercase letters denote aggregates, and superscript t indexes per-round (per-decision) quantities while parenthetical (t) indexes period-start state. Greek letters are mechanism parameters; the alphabet of design knobs is summarized in Table 1 below.

2.2 Voting power: two designs

Each agent has a *voting-power index* $W_i(t) \geq 0$ at period start, used for governance reporting. We consider two variants of CG-1:

Design A — Stake-only. Voting power is identified with stake:

$$W_i(t) := x_i(t).$$

The committed voting weight in a single round is $w_i^t := c_i^t$, where c_i^t is the agent's commitment that round. Voting history affects voting power only through stake updates.

Design B — Stake $\times$ reputation. Each agent carries an additional reputation account $r_i(t) > 0$, with $r_i(0) = 1$ and $\sum_{j \in N} r_j(t) = n$ enforced by renormalization (§2.7). Voting power is:

$$W_i(t) := x_i(t) \cdot r_i(t), \qquad w_i^t := c_i^t \cdot r_i(t).$$

Both designs satisfy $W_i(0) = x_i(0)$ — initial voting power proportional to stake — which directly implements the user's stated principle of stake-proportional voice. Design A keeps the model minimal; Design B adds a separate reputation channel that updates independently of stake on each round, providing a credibility-signaling mechanism that resists wealth-bias.

2.3 Proposals and outcomes

Each period t, a single proposal a_t enters the agenda. (Multi-proposal extension is straightforward but defers analysis.) Each proposal has a hidden type $\theta_t \in \{+, -\}$ drawn i.i.d. with prior $\mu = \Pr(\theta_t = +) \in (0, 1)$. If the proposal is accepted and executed, the realized change in pool value is:

$$\Delta X_t = \theta_t \cdot \rho \cdot X(t),$$

where $\rho \in (0, 1)$ is the per-proposal productivity scale. If the proposal is rejected, $\Delta X_t = 0$ by convention. We treat ρ as exogenous and constant for the formal analysis; endogenizing ρ via labor or information cost is left to paper 2.

The outcome θ_t is observable to the mechanism by time $t + \delta$ for some delay $\delta \geq 0$ measured against an agreed KPI. Our formal analysis takes $\delta = 0$; the delayed-observation extension is straightforward.

2.4 Information structure

Each agent receives a private signal $s_i^t \in \{+, -\}$ with accuracy:

$$\Pr(s_i^t = \theta_t \mid \theta_t) = q_i,$$

where the competence $q_i \in (1/2, 1)$ is the agent's signal accuracy. Signals are conditionally independent across agents given θ_t — the canonical assumption of weighted Condorcet jury theorems (Nitzan & Paroush 1982; Boland 1989). Generalizations to correlated signals are noted in §7.

The competence profile $(q_i)_{i \in N}$ is *persistent* (time-invariant) and treated as common knowledge for the formal analysis. The case where q_i is unknown or learned through observation is discussed in §7.

2.5 Action space and aggregation

In each period t, each agent simultaneously chooses: - Commitment $c_i^t \in [0, x_i(t)]$ — the stake she puts behind her vote; - Direction $y_i^t \in \{-1, +1\}$ — for or against the proposal.

Abstention is $c_i^t = 0$, in which case the direction is undefined and the agent does not contribute to the aggregation. The pair (c_i^t, y_i^t) is broadcast simultaneously; for on-chain implementations we recommend a commit–reveal protocol to enforce simultaneity (§2.10).

Define the *for-voter* and *against-voter* sets:

$$Y_t := \{i : y_i^t = +1, c_i^t > 0\}, \qquad Z_t := \{i : y_i^t = -1, c_i^t > 0\}.$$

The total committed weights are $W_Y^t = \sum_{i \in Y_t} w_i^t$ and $W_Z^t = \sum_{i \in Z_t} w_i^t$, and the total for-side commitment is $C_Y^t = \sum_{i \in Y_t} c_i^t$.

The aggregate vote is:

$$V_t := W_Y^t - W_Z^t.$$

2.6 Acceptance: three conjunctive gates

The proposal is *accepted* if and only if all three of the following hold: 1. **Aggregate threshold:** $V_t > Q$, where $Q \geq 0$ is a quorum parameter (default $Q = 0$ for simple weighted majority). 2. **Non-degeneracy:** $W_Y^t > 0$ (some agent is willing to vote for it). 3. **Coverage gate:** $C_Y^t \geq (1 - \zeta_-)\rho X(t)$ — the for-side commitment must cover the worst-case unshared loss.

The coverage gate is a feasibility primitive: it ensures that even if the proposal turns out unsuccessful and the loss falls on for-voters, no stake goes negative. Crucially, the gate scales with ζ_-: when $\zeta_- = 1$ (full burden-sharing), the gate is vacuous; when $\zeta_- = 0$ (pure for-voter loss-absorption), the gate enforces $C_Y^t \geq \rho X(t)$. We discuss the empirical binding rate in §6.

2.7 The four-case update rule

After acceptance, the outcome θ_t is realized and observable. Four cases govern the update:

Case C1 — Accepted and successful ($\theta_t = +$, $\Delta X_t = +\rho X(t)$). The pool gains. Stakes update as:

$$x_i(t+1) = x_i(t) + \zeta_+ \pi_i(t) \Delta X_t + (1 - \zeta_+) \cdot \mathbb{1}[i \in Y_t] \cdot \left[\eta \frac{c_i^t}{C_Y^t} + (1 - \eta) \frac{1}{|Y_t|} \right] \cdot \Delta X_t.$$

The first term is the universal dividend: ζ_+ fraction of the gain distributed pro-rata to all stakeholders. The second term is the skill premium: $(1-\zeta_+)$ fraction distributed only to for-voters, split between commitment-weighted (η) and flat-per-voter ($1-\eta$). For our recommended default $\eta = 1$ (pure c-share), the skill premium is proportional to commitment.

In Design B, the reputation update is multiplicative:

$$\tilde{r}_i(t+1) = \begin{cases} r_i(t)(1+\gamma_+) & \text{if } i \in Y_t \\ r_i(t)(1-\gamma_+) & \text{if } i \in Z_t \\ r_i(t) & \text{if } i \text{ abstained} \end{cases}$$

Case C2a — Accepted and unsuccessful ($\theta_t = -$, $\Delta X_t = -\rho X(t)$). The pool shrinks. Stakes update with the parallel structure:

$$x_i(t+1) = x_i(t) + \zeta_- \pi_i(t) \Delta X_t + (1-\zeta_-) \cdot \mathbb{1}[i \in Y_t] \cdot \left[\eta \frac{c_i^t}{C_Y^t} + (1-\eta) \frac{1}{|Y_t|} \right] \cdot \Delta X_t.$$

In Design B, reputation update is the role-swapped version:

$$\tilde{r}_i(t+1) = \begin{cases} r_i(t)(1-\gamma_-) & \text{if } i \in Y_t \\ r_i(t)(1+\gamma_-) & \text{if } i \in Z_t \\ r_i(t) & \text{if } i \text{ abstained} \end{cases}$$

The asymmetric Taleb case allows $\gamma_- > \gamma_+$; the symmetric default $\gamma_+ = \gamma_- = \gamma$ is analyzed in detail in §3 with quantitative bounds on permissible asymmetry.

Case C2b — Accepted and break-even ($\theta_t \in \{+,-\}$ but $\Delta X_t = 0$). Under the binary θ_t model this is a measure-zero event; no stake updates fire. If we generalize to continuous θ_t (paper 2 extension), C2b is handled with reputation updates only.

Case C3 — Rejected ($V_t \leq Q$, $W_Y^t \leq 0$, or coverage gate fails). No updates fire: stakes unchanged, reputation unchanged. The mechanism does not condition on counterfactual outcomes.

Reputation renormalization. After every period (C1 or C2a), apply:

$$r_i(t+1) := n \cdot \tilde{r}_i(t+1) / \sum_{j \in N} \tilde{r}_j(t+1).$$

This ensures $\sum_i r_i(t) = n$ at all t, placing the reputation process on a simplex (Pemantle 2007; Athreya & Karlin 1968).

2.8 Lemma 1 — Pool conservation

Lemma 1. Under the update rule of §2.7 with coverage gate satisfied, the pool obeys:

$$X(t+1) = X(t) + \Delta X_t = (1 + \theta_t \rho \mathbb{1}[\text{accepted}]) X(t),$$

and all individual stakes remain non-negative: $x_i(t+1) \geq 0$ for all $i \in N$.

Proof. For Case C1 and C2a, the share weights satisfy:

$$\sum_{i \in N} \zeta \pi_i + (1-\zeta) \sum_{i \in Y_t} \left[\eta \frac{c_i^t}{C_Y^t} + (1-\eta) \frac{1}{|Y_t|} \right] = \zeta + (1-\zeta)(\eta + (1-\eta)) = 1,$$

so $\sum_i \Delta x_i = \Delta X_t$, and $X(t+1) = X(t) + \Delta X_t$. For non-negativity in C2a, the largest possible loss for $i \in Y_t$ is:

$$|\Delta x_i| = \left| \zeta_- \pi_i \Delta X_t + (1-\zeta_-) \left[\eta \frac{c_i^t}{C_Y^t} + \frac{1-\eta}{|Y_t|} \right] \Delta X_t \right|.$$

Under the coverage gate $C_Y^t \geq (1-\zeta_-)\rho X(t)$, the for-voter contribution is bounded by c_i^t for $\eta = 1$, and pool conservation guarantees the dividend portion does not over-deplete. Details in Appendix A.1. □

2.9 Success metric: S1 and S2

We compare two success-classification metrics:

S_1 **— Pure growth:** Successful $\iff \Delta X_t > 0$.

S_2 **— Growth with equity:** Successful $\iff \Delta X_t > 0$ AND $\bar{\Delta G}_t \leq \varepsilon_G$, where $\bar{\Delta G}_t$ is an exponentially-weighted moving average (EWMA) of the Gini change:

$$\bar{\Delta G}_t = \lambda \Delta G_t + (1-\lambda)\bar{\Delta G}_{t-1}, \qquad \lambda = 1 - 2^{-1/H}.$$

H is the EWMA half-life (in proposals); ε_G is the equity-violation threshold.

S_2 provides protection against rent-extracting proposals that grow the pool but concentrate the gain. We prove in §5 that S_2 caps coalition extraction by $O(\sqrt{\varepsilon_G \lambda})$. The default values for our recommended mechanism are $H = 50$ and $\varepsilon_G = 0.003$.

2.10 Participation incentive, decay, and commit–reveal

Participation tie-breaker. To rule out the all-abstain equilibrium, the mechanism rewards active voters with a small per-round bonus:

$$\text{bonus}_i = \varepsilon \cdot w_i^t, \qquad \varepsilon = \kappa \rho^2, \quad \kappa = 0.2.$$

We fix $\kappa = 0.2$ throughout; the empirical validation §6 confirms $\kappa = 0.2$ is small enough not to distort equity.

PR-c funding (formal specification). A fraction $f_{\text{PR-c}} \in [0,1]$ of the per-period slashing pool is recycled into the participation bonus reserve. Specifically: in C2a rounds, where for-voters absorb total loss $L_t = (1-\zeta_-)\rho X(t)$, we divert $f_{\text{PR-c}} \cdot L_t$ to a participation-bonus pool $P(t)$:

$$P(t+1) = P(t) + f_{\text{PR-c}} \cdot L_t - \sum_i \text{bonus}_i^t.$$

The bonus pool is drained pro-rata across participating agents. If the pool runs dry, bonuses are scaled down. Default: $f_{\text{PR-c}} = 0.05$ (5% of slash recycled). Pool conservation holds in expectation; in deficit periods the bonus is reduced, in surplus periods the bonus is at the recommended level.

Optional decay τ. For long-horizon resilience we allow a small per-round decay toward the per-capita mean:

$$x_i(t+1) \leftarrow (1-\tau)x_i(t+1) + \tau \bar{x}(t+1),$$

where $\bar{x} = X/n$. The decay $\tau \in [0,1)$ is a Harberger-style anti-hoarding parameter (Posner & Weyl 2018), set to 0 in the default but configurable.

Commit–reveal protocol. For on-chain implementations where commitments are observable through the mempool, simultaneity is enforced by a two-phase protocol: agents first broadcast cryptographic hashes $H(c_i^t, y_i^t, \text{nonce}_i)$, then in a subsequent reveal phase publish $(c_i^t, y_i^t, \text{nonce}_i)$. The mechanism verifies hashes before computing V_t. Under the recommended $(\zeta_+, \zeta_-) = (1, 0)$ Taleb default, there is no positive-round arms race because gains are universal; commit–reveal primarily defends against last-minute reduction of c_i on observed-failing proposals (§7).

2.11 Parameter glossary

Symbol	Role	Default
n	agent count	—
$X(t)$	pool total	$X(0) > 0$ given
ρ	productivity scale	$\rho = 0.05$
μ	prior on $\theta_t = +$	$\mu = 0.5$
q_i	competence	heterogeneous
(c_i^t, y_i^t)	commitment and direction	—
$\zeta_+ \in [0, 1]$	growth pro-rata fraction	**1** (Taleb-asymmetric)
$\zeta_- \in [0, 1]$	loss pro-rata fraction	**0** (Taleb-asymmetric)
$\eta \in [0, 1]$	aligned-set split	**1** (pure c-share)
γ_+, γ_-	reputation rates	$\gamma_+ = \gamma_- = 0.05$
$w_{\max}$	Lorenz cap	1 (none)
τ	decay rate	0
κ	participation scaling	**0.2** (standardized)
$\varepsilon = \kappa\rho^2$	participation bonus rate	0.0005 at $\rho = 0.05$
$f_{\text{PR-c}}$	slash recycle fraction	0.05
Q	quorum threshold	0
H	EWMA half-life (S_2)	50
ε_G	Gini-change threshold (S_2)	0.003

The **CG-1 default** mechanism uses Design B, $(\zeta_+, \zeta_-) = (1, 0)$, $\eta = 1$, symmetric γ, S_2 success, PR-c-funded ε, no Lorenz cap, no decay, $Q = 0$, with the coverage gate scaled to ζ_-.

---

3. Strategic Analysis

3.1 Utility specification

Each agent maximizes the expected discounted log of her voting-power index:

$$U_i = \mathbb{E}\left[\sum_{t \geq 0} \beta^t \log W_i(t)\right],$$

with discount factor $\beta \in (0, 1)$. In Design A, $\log W_i(t) = \log x_i(t)$ — the canonical Kelly criterion. In Design B, $\log W_i(t) = \log x_i(t) + \log r_i(t)$ — log of stake plus log of reputation. Renormalization (§2.7) keeps $\log r_i$ bounded above by $\log n$, preventing the unbounded-utility pathology.

This utility is the natural choice for percentage-return dynamics; it yields Kelly-style commitments in equilibrium (Kelly 1956; Cover & Thomas 2006, Ch. 16). CRRA generalizations preserve the qualitative results; we discuss in Appendix B.

Grounding note. The utility-on-voting-power-index formulation places direct utility weight on reputation r_i, not only on stake x_i. This is sometimes questioned: *why would a rational agent value reputation for its own sake when payouts in our recommended default are c-share (commitment-proportional, not reputation-proportional)?*

The formulation is justified empirically by direct industry inspiration (cf. §1.6½): production governance designs such as the **Power Protocol** treat the reputation account as a *first-class economic object* — agents wager, accrue, and lose reputation in ways that map onto observable in-protocol consequences for their future ability to influence resource allocation. Under such designs, the utility weight on reputation is not a modelling postulate but an observed behavioural fact: agents demonstrably trade off stake against reputation in equilibrium because reputation determines their share of future allocation decisions.

Paper 1 adopts the *non-transferable* reputation model — reputation amplifies voting weight in the period's aggregation but is not itself tradeable. This is the cleanest setting in which to prove the impossibility result (Theorem 7) and characterize the parametric family. The *transferable / tokenized* reputation case — where the utility weight on reputation becomes literal (market-priced), with reputation markets, delegation, and tokenized influence — is the subject of paper 2 and is the more direct match to current production governance protocols.

Readers concerned with the empirical realism of the assumption may treat $\log r_i$ in the utility as a proxy for the *shadow price* of reputation in a tokenized-reputation extension; the formal analysis of paper 1 does not depend on whether this shadow price is materialized as a market price or remains in-protocol.

3.2 Sincere strategy

A *sincere strategy* is: - **Direction:** $y_i^t = s_i^t$ — vote according to your signal. - **Commitment:** $c_i^t = c^*(s_i^t, x_i(t), q_i, \text{public history})$, a Kelly-like fractional commitment derived from the posterior on θ_t and the perceived equilibrium aggregate.

In the symmetric homogeneous case ($q_i = q$ for all i, $\mu = 1/2$), the sincere Kelly commitment in equilibrium has the closed form:

$$c_i^* = (2q - 1) \cdot x_i \cdot K(\rho, n),$$

where $K(\rho, n) \in (0, 1]$ is the symmetric BNE Kelly multiplier solving a fixed-point equation involving the aggregate for-side commitment (derivation in Appendix A.2).

3.3 Theorem 2 — Direction-DS, corrected for the (1, 0) corner

Critical clarification. Our recommended default uses $(\zeta_+, \zeta_-) = (1, 0)$. Under this setting:

- **C1 (accepted + successful):** the entire pool gain is distributed as a *universal dividend* pro-rata to all stake-holders, regardless of vote direction. The for-voter alignment advantage on the stake channel is therefore **zero**.
- **C2a (accepted + unsuccessful):** only for-voters absorb the loss. Against-voters and abstainers keep their stakes.

This is a substantive change from the $(\zeta_+, \zeta_-) = (0, 0)$ baseline analysis. Under (1, 0), the *stake-channel incentive to vote sincerely is non-positive*: an agent who votes for what turns out to be a bad proposal loses stake, while voting against (or abstaining) avoids this loss. So Design A under (1, 0) has no stake-channel skin-in-the-game and is degenerate — the all-abstain or always-against equilibrium dominates.

Theorem 2 (Direction-DS in Design B under (1, 0)).

Assume Design B (stake × reputation) with symmetric reputation rates $\gamma_+ = \gamma_- = \gamma > 0$. Conditional on participation ($c_i^t > 0$) and non-pivotality, sincere direction $y_i^t = s_i^t$ weakly dominates $y_i^t = -s_i^t$ provided:

$$\boxed{\gamma > \frac{\alpha(1-q)\rho}{4(q-1/2)n_{\text{eff}}}} \tag{1}$$

where α is the per-period acceptance probability, q is the agent's competence (or the population mean $\bar{q}$ in the symmetric case), and $n_{\text{eff}} = (1-\bar{q})n$ is the expected wrong-for-voter set size on negative rounds.

Under default parameters ($\alpha \approx 0.5$, $\rho = 0.05$, $q = 0.7$, $n = 100$, hence $n_{\text{eff}} = 30$), the right-hand side equals 0.0003, so any $\gamma \geq 0.0003$ suffices. Our default $\gamma = 0.05$ satisfies the condition by factor 167.

Proof. Under sincere play with signal $s_i = +$ and the (1, 0) corner, conditioning explicitly on $s_i = +$:

Stake channel. By Bayes' rule with prior $\mu = 1/2$, $\Pr(\theta = + \mid s_i = +) = q_i$ and $\Pr(\theta = - \mid s_i = +) = 1 - q_i$. The expected per-period $\Delta \log x_i$ for sincere voting ($y_i = +$ given $s_i = +$), conditioned on acceptance and on signal:

$$\mathbb{E}^{\text{sincere}}[\Delta \log x_i \mid s_i = +] = \alpha \cdot [q_i \cdot 0 + (1 - q_i)\log(1-g)]\,,$$

where $g = \rho \cdot c_i^t / C_Y^t \approx \rho / n_{\text{eff}}$ in the symmetric early-time regime. The $q_i \cdot 0$ term arises because on C1 ($\theta = +$) all stake-holders receive the universal dividend pro-rata, so the for-voter has no extra stake gain. The $(1 - q_i)\log(1-g)$ term is the C2a loss when the proposal turned out destructive. For misvote ($y_i = -$ given $s_i = +$), the agent is in Z_t and stake is unchanged on both C1 and C2a:

$$\mathbb{E}^{\text{misvote}}[\Delta \log x_i \mid s_i = +] = 0.$$

Hence the *stake-channel disadvantage* of sincere voting is:

$$\Delta_{\text{stake}} = \mathbb{E}^{\text{sincere}}[\Delta \log x_i \mid s_i = +] - \mathbb{E}^{\text{misvote}}[\Delta \log x_i \mid s_i = +] = \alpha(1 - q_i)\log(1-g) < 0.$$

To first order in small g: $\Delta_{\text{stake}} \approx -\alpha(1 - q_i)g$.

Reputation channel. The renormalization preserves the cross-agent ordering; the per-period log-reputation update for sincere voting is:

$$\mathbb{E}^{\text{sincere}}[\Delta \log r_i] = q_i \log(1+\gamma) + (1 - q_i)\log(1-\gamma) \approx (2q_i - 1)\gamma.$$

For misvoting:

$$\mathbb{E}^{\text{misvote}}[\Delta \log r_i] = q_i \log(1-\gamma) + (1 - q_i)\log(1+\gamma) \approx -(2q_i - 1)\gamma.$$

Hence the *reputation-channel advantage* of sincere voting is:

$$\Delta_{\text{rep}} \approx 2(2q_i - 1)\gamma = 4(q_i - 1/2)\gamma.$$

Combined utility (Design B: $U_i = \log x_i + \log r_i$). Sincere direction is dominant iff $\Delta_{\text{stake}} + \Delta_{\text{rep}} \geq 0$.

Necessary and sufficient condition (first-order in small g, γ). The exact necessary-and-sufficient condition retains the log structure:

$$\boxed{4(q_i - 1/2)\gamma + \alpha(1 - q_i)\log(1 - g) \geq 0.} \tag{NS}$$

This holds iff $\gamma \geq \frac{\alpha(1-q_i)|\log(1-g)|}{4(q_i-1/2)}$. For small g, $|\log(1-g)| \approx g$, recovering:

$$\gamma \geq \frac{\alpha(1 - q_i)g}{4(q_i - 1/2)}.$$

Sufficient condition (the boxed condition (1)). Substituting $g \approx \rho/n_{\text{eff}}$ and bounding gives the more conservative form in equation (1). Under default parameters ($\alpha \approx 0.5$, $\rho = 0.05$, $q = 0.7$, $n = 100$, hence $n_{\text{eff}} = 30$), the NS condition gives $\gamma \geq 0.00021$ (versus 0.0003 for the sufficient form). Our default $\gamma = 0.05$ satisfies both by factor 167–238. □

Design A under (1, 0): a conditional statement. In Design A, there is no reputation channel ($\Delta_{\text{rep}} = 0$), so the combined utility is $\Delta_{\text{stake}} = -\alpha(1 - q_i)g < 0$ — sincere direction strictly *underperforms* misvote on the stake channel. However, abstention (vote 0) forgoes the participation bonus εw_i that PR-c provides to active voters. The net comparison is:

$$\text{Sincere vs abstain advantage} = \varepsilon w_i - \alpha(1 - q_i)g.$$

Under default parameters ($\varepsilon = 0.0005$, $\alpha = 0.5$, $q = 0.7$, $g \approx \rho/n_{\text{eff}} \approx 0.0017$): sincere-vote advantage is $\varepsilon w_i - 0.00026 w_i \approx 0.00024 w_i > 0$. So **PR-c rescues sincere voting in Design A under (1, 0)** — but the result is fragile: a 2× higher ρ or 2× lower PR-c funding flips the sign. The correct claim is:

> *In Design A under (1, 0), sincere voting requires $\varepsilon > \alpha(1-q_i)g$. Under default PR-c the condition holds, but the margin is thin and the result is non-robust to parameter shifts.*

Design B with (1, 0) provides a much wider margin (factor 167–238 above the NS threshold), which is the operational reason we recommend it.

3.4 Theorem 3 — BNE existence

> **Theorem 3 (Sincere-Kelly BNE).** *Assume: (i) Symmetric setting: $q_i = q \in (1/2, 1)$ for all i; $\mu = 1/2$. (ii) Interior Kelly: $\rho < \rho^*(q, n) := (2q - 1)/n$. (iii) Participation tie-breaker $\varepsilon > 0$ (§2.10) rules out the all-abstain equilibrium. Then a symmetric sincere-Kelly profile is a Bayesian Nash equilibrium of CG-1. Under heterogeneous q_i, existence follows from Glicksberg (1952); uniqueness fails in general.*

Proof sketch. Existence on the symmetric simplex follows from Brouwer's fixed point theorem applied to the best-response map; continuity holds under (i)–(iii). Uniqueness on the *interior* (ruling out the all-in corner) follows from the monotonicity of the symmetric best-response and the strict concavity of log-utility. Multiplicity outside the hypotheses includes: the all-in corner equilibrium (overcomes by interior Kelly assumption); the all-abstain equilibrium (overcomes by $\varepsilon > 0$); and asymmetric equilibria in heterogeneous-q environments (which are not symmetric BNE but may be Glicksberg BNE).

Under heterogeneous q_i, the strategy space $\prod_i [0, x_i(0)] \times \{-1, +1\}$ is compact and convex (after randomization on direction), and payoffs are continuous in strategies; Glicksberg (1952) applies. Tullock-with-budget literature (Cornes & Hartley 2005; Konrad 2009) gives multiplicity examples. Full proof in Appendix A.4. □

3.5 The two design variants in equilibrium

The (1, 0) analysis above isolates a sharp contrast:

- **Design A under (1, 0)** has no stake-channel skin-in-the-game on C1 (the universal dividend equalizes for- and against-voters), and the C2a loss is concentrated on for-voters. Sincere voting requires the PR-c participation bonus to exceed the expected C2a loss; this condition holds under default parameters by a thin margin ($\varepsilon w_i - \alpha(1-q_i)g \approx +0.00024 w_i$, see §3.3 boxed conditional), but the margin is fragile to parameter shifts. We therefore do *not* recommend Design A with $\zeta_+ > 0$ as the operating point — Design B provides a much wider margin (factor 167–238 above the NS threshold).
- **Design B under (1, 0)** is the recommended default. The reputation channel provides the missing skin-in-the-game on the upside: agents are rewarded with future voting power when they vote correctly, even though their stake is not directly rewarded on C1. Under the boxed condition (1), this is sufficient for direction-DS with a wide robustness margin.

This dichotomy clarifies an asymmetry in the family: the (1, 0) corner is *robustly viable* only in Design B. For Design A, the operational alternative is $(\zeta_+, \zeta_-) = (0, 0)$ — the pure-skin-in-the-game baseline that suffers maximally from the impossibility T7. The intermediate $(0.5, 0)$ corner trades these effects.

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4. Dynamics and Impossibility

4.1 Proposition 4 — Early-time pool growth

Proposition 4 (Pool growth under bounded weight ratios). *Assume: (i) Theorem 3 conditions (symmetric homogeneous q, $\mu = 1/2$, $\rho < \rho^*$, $\varepsilon > 0$, $W_Y^t > 0$). (ii)* ***Bounded weight ratio hypothesis:*** *$\sup_t \max_{i,j} w_i^t / w_j^t \le R$ for some $R < \infty$, where the supremum is over committed voters in period t. (iii) Conditionally independent signals: $\Pr(s_i^t = \theta_t \mid \theta_t) = q_i$ independently across i.*

Then in the early-time regime, the per-period probability of correct acceptance satisfies (Nitzan & Paroush 1982; Boland 1989):

$$\phi := \Pr(\theta = + \mid \text{accepted}) \ge 1 - \exp(-c(q_{\min}, R) \cdot n)\,,$$

where $c(q_{\min}, R) > 0$ depends on the minimum competence $q_{\min}$ and the weight ratio R. Specifically:

$$c(q_{\min}, R) = \frac{(2q_{\min} - 1)^2}{2R^2}.$$

Hence the expected per-period log-growth of the pool satisfies:

$$\mathbb{E}[\log X(t+1) - \log X(t)] \ge \alpha \cdot [(1 - e^{-cn}) \log(1+\rho) + e^{-cn} \log(1-\rho)] > 0$$

for n sufficiently large.

Proof sketch. Standard weighted Condorcet jury theorem (Nitzan & Paroush 1982 Theorem 1; Boland 1989 Corollary 3.1) under bounded weight ratio R gives the exponential rate of correct decision in n. The constant c derives from the second-moment bound on $V_t / \sum w_i$ via Hoeffding's inequality applied to the weighted sum of i.i.d. signals. Full derivation in Appendix A.5. □

Note on the bounded-weight hypothesis. Hypothesis (ii) is exactly the condition that fails asymptotically under heterogeneous competence (per Theorem 7): once reputation concentrates, $\max w_i / \min w_i \to \infty$ and the Condorcet bound degrades. Proposition 4 holds in the

early-time regime before this concentration, which empirically corresponds to $t \lesssim T_*$ where T_* depends on q-spread (see §6.7 for the empirical T_*).

Tightness of the rate constant. The constant $c(q_{\min}, R) = (2q_{\min}-1)^2/(2R^2)$ is a Hoeffding bound and is not tight. For symmetric homogeneous q with $R = 1$, the sharper Nitzan–Paroush (1982) rate is $-\log[2\sqrt{q(1-q)}]$, which is larger than $(2q-1)^2/2$ for q in the relevant range. We use the Hoeffding rate here because it generalizes cleanly to arbitrary R; the symmetric special case admits the sharper bound.

4.2 Theorem 5 — Reputation concentration via SA + entropy Lyapunov

The renormalized reputation process under sincere play tracks a replicator ODE on the simplex with strictly ordered drifts. We give two related results: **exponential ergodicity to an $O(\gamma)$-neighborhood of the highest-competence vertex** under our recommended *constant* step size γ, and **a.s. convergence to the vertex** under a decreasing schedule. The constant-γ version is what is operationally used; the decreasing-γ_t version is included to clarify the asymptotic statement.

> **Theorem 5a (Concentration under constant step size; main).** *Let $q_i \in [q_{\min}, q_{\max}]$ be persistent and strictly heterogeneous: $q_{\max} > q_{\min} \geq 1/2 + \nu$ for some $\nu > 0$. Under sincere play in CG-1 Design B with renormalized reputation ($\sum_i r_i(t) = n$) and small constant reputation rate $\gamma_+ = \gamma_- = \gamma$, the reputation process is exponentially ergodic with a stationary distribution π_γ concentrated in an $O(\gamma)$-neighborhood of the highest-competence vertex e_{i^*}:*
>
> $$\mathbb{E}_{r\sim\pi_\gamma}[r_{i^*}/n] = 1 - O(\gamma), \qquad i^* := \arg\max_i q_i.$$
>
> *Moreover, for any $\delta > 0$, $\Pr_{\pi_\gamma}(r_{i^*}/n < 1-\delta) \leq Ce^{-\delta^2/\gamma}$ for some constant C depending on q-spread.*
>
> **Theorem 5b (Almost-sure convergence under decreasing step size).** *If γ is replaced by a sequence $\gamma_t \to 0$ satisfying $\sum_t \gamma_t = \infty$ and $\sum_t \gamma_t^2 < \infty$ (Robbins–Monro conditions), then $r_{i^*}(t)/n \to 1$ almost surely.*

Proof of Theorem 5a.

Step 1 — Mean-field drift on the simplex. As before, the renormalized reputation process satisfies

$$\mathbb{E}[r_{t+1} - r_t \mid r_t] = \gamma \cdot r_t \odot \left(R(q) - \frac{\langle r_t, R(q)\rangle}{n} \right) + O(\gamma^2),$$

where $R(q_i)$ is the expected single-step log-fitness defined in equation (3).

Step 2 — Entropy Lyapunov function. Define $L(r) := -\log(r_{i^*}/n) \geq 0$, with $L(r) = 0$ iff $r = ne_{i^*}$. Along the replicator ODE $\dot r = r \odot (R(q) - \langle r, R(q)\rangle/n)$:

$$\dot L = \langle r, R(q)\rangle/n - R(q_{i^*}) \leq 0,$$

with equality only at the target vertex. L is a strict Lyapunov function.

Step 3 — Foster–Lyapunov drift condition. The discrete process satisfies a Foster–Lyapunov drift inequality:

$$\mathbb{E}[L(r_{t+1}) - L(r_t) \mid r_t] \leq -c(r_t)\gamma + C_2\gamma^2,$$

where $c(r) > 0$ on the simplex interior away from the boundary. The drift condition is uniformly negative outside a *petite set* — a compact neighborhood of the highest-competence

vertex on which the chain returns reliably under the multiplicative-update dynamics. (The boundary of the simplex, where the replicator vector field vanishes, is a measure-zero set under the noisy update, so the chain spends asymptotically zero time near it.) By the theory of V-uniformly ergodic Markov chains (Meyn & Tweedie 2009, Ch. 14–15), this drift-and-petite-set condition implies exponential ergodicity: there exists a unique stationary distribution π_γ and constants $\rho < 1$, $M < \infty$ such that

$$\|P^t(r, \cdot) - \pi_\gamma\|_{TV} \leq M\rho^t(1 + L(r)).$$

Step 4 — Stationary distribution location. The fixed-point analysis: at stationarity, $\mathbb{E}_{\pi_\gamma}[L(r)] \leq C_2\gamma/c$, where c is the minimum drift coefficient on a compact subset of the simplex interior. Hence the stationary distribution concentrates within $O(\gamma)$ of the target vertex in entropy distance, which translates to $\mathbb{E}_{\pi_\gamma}[r_{i^*}/n] \geq 1 - O(\gamma)$.

Step 5 — Exponential tail bound. The exponential tail $\Pr(r_{i^*}/n < 1 - \delta) \leq Ce^{-\delta^2/\gamma}$ follows from a sub-Gaussian concentration argument under the bounded-step-size noise (Boucheron, Lugosi & Massart 2013, Ch. 6). □

Proof of Theorem 5b. Under $\gamma_t \to 0$ with $\sum \gamma_t = \infty$, $\sum \gamma_t^2 < \infty$, the Robbins–Siegmund supermartingale convergence theorem applies cleanly: $L(r_t)$ is a non-negative almost-supermartingale with summable error, so $L(r_t)$ converges a.s. to a non-negative random variable; combined with the strict Lyapunov property and $\sum \gamma_t = \infty$, the limit is a.s. zero (Borkar 2008, Ch. 2). □

Practical interpretation. Under our recommended constant $\gamma = 0.05$, the reputation does not converge a.s. to the vertex — it fluctuates within an $O(\gamma) = O(0.05)$ neighborhood. The empirical Figure 1 trajectory at high T shows this fluctuation: r_{i^*}/n hovers around 0.95–1.00, not at exactly 1. For practical equity claims (Gini below threshold over governance-realistic horizons), this is more than adequate. The decreasing-γ_t schedule (Theorem 5b) is included to make the asymptotic concentration mathematically clean.

Tie-breaking: when multiple q_i achieve the maximum, the stationary distribution is concentrated near the convex hull of optimal vertices.

4.3 Lemma 6 — Stake inheritance

> **Lemma 6 (Stake follows reputation, with rate).** *Under Theorem 5 conditions and c-share payouts ($\eta = 1$) with the $(0, 0)$ baseline, conditional on reputation having concentrated such that $r_{i^*}/n \geq 1 - \delta$ for δ small, the expected per-period log-stake drift gap between i^* and any other agent j is bounded above by:*
>
> $$\mathbb{E}[\Delta \log x_{i^*} - \Delta \log x_j] \geq c_0(1 - \delta)\rho \log\left(\frac{q_{i^*}(1 - q_j)}{(1 - q_{i^*})q_j}\right) + O(\rho^2)$$
>
> *for some constant $c_0 > 0$. Hence stake concentrates on i^* asymptotically.*

Proof sketch. Under near-full reputation concentration on i^*, i^*'s voting weight $w_{i^*} = c_{i^*}r_{i^*}$ dominates V_t. Acceptance probability conditional on $\theta = +$ approaches q_{i^*} (when i^* aligns with truth); conditional on $\theta = -$ approaches $1 - q_{i^*}$. For any other agent j, her acceptance-conditioned alignment is q_j. The log-likelihood-ratio difference in alignment probabilities gives the bound. Full proof in Appendix A.7. □

We give an explicit rate: the c-share rate is $\Theta(\rho \log[q_{i^*}/q_j])$ per round, vs. w-share which is $\Theta(\rho)$ per round — confirming c-share is asymptotically slower.

4.4 Theorem 7 — Impossibility, restricted to Design B

Theorem 7 (Impossibility for Design B with c-share). *Let $\mathcal{M}_B$ be the class of CG-1 Design B mechanisms with c-share payouts ($\eta = 1$) and renormalized reputation ($\sum_i r_i(t) = n$), satisfying:*

(P5) ***Strict reward asymmetry on the combined channel.*** *The reward is measured on $W_i = x_i \cdot r_i$ (the voting-power index), NOT on stake alone. Specifically: for accepted productive proposals, $\mathbb{E}[\Delta \log W_i \mid i \in Y_t, \theta = +] > \mathbb{E}[\Delta \log W_i \mid i \in Z_t, \theta = +]$.*

Note on stake-only formulation. Under our recommended $(\zeta_+, \zeta_-) = (1, 0)$ default, the stake-channel reward is identical for aligned and misaligned voters on C1 (universal dividend). (P5) is satisfied on the *combined* channel via the reputation update, but it would not hold if restricted to stake alone.

(P6) Reward weakly increasing in commitment.

Assume $q_i \in [q_{\min}, q_{\max}]$ persistent and strictly heterogeneous, $q_{\min} > 1/2$.

Then no mechanism $M \in \mathcal{M}_B$ satisfies (O2) $\mathbb{E}[G(t+1) \mid \mathcal{F}_t] \leq G(t)$ under sincere BNE; furthermore, $\pi_{i^*}(t) \to 1$ a.s. as $t \to \infty$.

Proof. By Theorem 5, $r_{i^*}/n \to 1$ a.s. By Lemma 6, conditional on reputation concentration, stake concentrates on i^* as well: $\pi_{i^*}(t) \to 1$ a.s. Hence $G(t) \to 1$, contradicting bounded $\mathbb{E}[G(t+1) \mid \mathcal{F}_t]$. □

Open problem (Design A). Whether T7 holds for Design A with $(\zeta_+, \zeta_-) = (0, 0)$ and c-share payouts is an open problem. The Athreya–Karlin urn argument does not apply cleanly because the c-share update is additive in stake, not multiplicative, and the standard Pólya-urn embedding is non-trivial. Intuitively the same conclusion should hold — high-competence agents accumulate stake via more frequent C1 wins — but a rigorous proof requires either a different machinery (e.g., a direct martingale argument on the stake process) or an embedding into a known concentration result. We leave this for future work. The recommended default uses Design B, which Theorem 7 covers.

4.5 Proposition 8 — Taleb-corner rate

Proposition 8 (Taleb-corner inequality growth bound). *Under $(\zeta_+, \zeta_-) = (1, 0)$ with c-share payouts ($\eta = 1$), the expected per-period log-stake drift gap between agents i and j with $q_i > q_j$ is bounded by:*

$$\Delta_{ij} := \mathbb{E}[\Delta \log x_i - \Delta \log x_j] \leq (1 - \mu)(q_i - q_j) \cdot \frac{\rho}{(1 - \bar{q})n}, \tag{5}$$

in the early-time regime where the wrong-for set has expected size $\mathbb{E}[|Y_t \mid \theta_t = -|] \approx (1 - \bar{q})n$.

Cumulative log-inequality after T periods is bounded:

$$\mathbb{E}[|\log \pi_i(T) - \log \pi_j(T)|] = O\left(\frac{T\rho(q_i - q_j)}{n_{\text{eff}}}\right), \qquad n_{\text{eff}} := (1 - \bar{q})n.$$

Proof sketch. Under $(1, 0)$: positive rounds distribute $\rho X(t)$ pro-rata to all stake-holders (the universal dividend), leaving relative shares unchanged. Only negative accepted rounds contribute to inequality dynamics.

On a negative round, only for-voters $Y_t^- := \{i \in Y_t : \theta_t = -\}$ lose stake. Under c-share, the per-agent loss share is $c_i^t / C_Y^t \approx 1/|Y_t^-|$ in the symmetric early-time regime. For agent i, the probability of being in Y_t^- (signal was wrong) is $1 - q_i$. The expected log-stake loss for i on a negative accepted round is:

$$\mathbb{E}[\Delta \log x_i \mid \theta = -, \text{accept}] \approx -(1 - q_i)\rho \cdot \mathbb{E}[1/|Y_t^-|].$$

By Jensen's inequality, $\mathbb{E}[1/|Y_t^-|] \geq 1/\mathbb{E}[|Y_t^-|] = 1/((1 - \bar{q})n)$ in the symmetric case.

The drift gap is:

$$\begin{aligned}\Delta_{ij} &\approx (1 - \mu)[\mathbb{E}[\Delta \log x_j \mid \theta = -, \text{accept}] - \mathbb{E}[\Delta \log x_i \mid \theta = -, \text{accept}]] \\ &= (1 - \mu)(q_i - q_j)\rho \cdot \mathbb{E}[1/|Y_t^-|] \\ &\leq (1 - \mu)(q_i - q_j)\rho/((1 - \bar{q})n).\end{aligned}$$

Cumulative bound by linearity. Full proof in Appendix A.8. □

Numerical example. At $n = 100$, $\rho = 0.05$, $\bar{q} = 0.7$, $\mu = 1/2$, and competence spread $q_i - q_j = 0.2$: per-period drift gap is $\Delta_{ij} \leq 0.5 \cdot 0.2 \cdot 0.05/(0.3 \cdot 100) \approx 1.7 \times 10^{-4}$. Cumulative log-inequality over 500 periods is bounded by ~ 0.083, corresponding to a Gini below ≈ 0.05 in a log-normal approximation — matching the empirical 0.039 we observe in §6.

This rate explains why the Taleb corner is *practically* equity-preserving on governance horizons: the drift coefficient scales as $1/n_{\text{eff}}$, and for $n_{\text{eff}} = O(n)$ in the early time regime, governance-realistic horizons ($T \sim 10^2$ to 10^3) fall well within the regime where the asymptotic concentration has not yet manifested.

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5. Bribery and Rent Extraction

This section develops the coalition-extraction bound under the S_2 success metric. We first formalize two threat models, then prove three supporting lemmas, and finally state the main bound (Theorem 12). The bound demonstrates that the bribery-resistance of CG-1 is *tunable* via two parameters (H, ε_G).

5.1 Threat models

Threat model B1 — Internal rent extraction. A coalition $C \subseteq N$ collectively proposes a transaction that transfers value from $N \setminus C$ to C, dressed as a productive proposal. Examples: (i) a treasury allocation to a coalition member's external venture with low realized return; (ii) a fee-structure change that asymmetrically benefits the coalition; (iii) a quality-grading change that re-weights existing assets toward the coalition.

Threat model B2 — Vote-trading. Independent agents i, j agree to vote for each other's preferred proposals across multiple rounds, increasing combined acceptance frequency for proposals serving narrow interests. Under closed-economy paper 1 scope, this manifests as correlated voting patterns rather than explicit side payments.

Both models are subsumed by the general problem: bound the per-period rent extraction by any coalition under the chosen success metric and parameter setting.

5.2 Lemma 9 — Gini sensitivity to mass transfers

Lemma 9 (Gini sensitivity). *For a coalition C of size k that captures a fraction $\sigma \in [0,1]$ of the period's pool flow to the exclusion of $N \setminus C$, the resulting change in Gini coefficient satisfies:*

$$\Delta G \geq \alpha(k,n) \cdot \sigma + O(\sigma^2), \quad \alpha(k,n) := 1 - 2k/n.$$

In particular, for small $k \ll n$: $\alpha(k,n) \approx 1$, so $\Delta G \approx \sigma$.

Proof sketch. Express Gini via the Lorenz curve: $G(L) = 1 - 2\int_0^1 L(u)du$ where L is the Lorenz function. A mass transfer of σ from the lower $1-k/n$ fraction to the upper k/n fraction shifts the Lorenz curve downward by approximately σ on the bottom $1-k/n$ portion, giving a Gini change $\Delta G \approx (1-k/n)\sigma + (k/n)\cdot 0 \approx (1-2k/n)\sigma + O(\sigma^2)$. Formal version via Hardy-Littlewood-Pólya majorization (Marshall, Olkin & Arnold 2011, Ch. 1). Full derivation in Appendix A.9. □

5.3 Lemma 10 — EWMA filter steady-state bound

Lemma 10 (EWMA-Gini steady-state constraint). *If $\bar{\Delta G}_t \leq \varepsilon_G$ for all t where $\bar{\Delta G}_t = \lambda \Delta G_t + (1-\lambda)\bar{\Delta G}_{t-1}$ with $\lambda \in (0,1)$, then for any window $[t_0, t_0+T]$ with $T \geq 1/\lambda$:*

$$\frac{1}{T}\sum_{t=t_0}^{t_0+T-1} \Delta G_t \leq \varepsilon_G + O(\lambda).$$

Proof. The EWMA is a stable IIR filter. The DC gain (steady-state response to a constant input) is 1, so the time-average of ΔG_t equals the steady-state $\bar{\Delta G}$. The constraint $\bar{\Delta G}_t \leq \varepsilon_G$ at all t implies the steady-state $\bar{\Delta G} \leq \varepsilon_G$. For finite T, transient effects contribute $O(\lambda^T)$, which is bounded by $O(\lambda)$ for $T \geq 1/\lambda$. Full derivation in Appendix A.10. □

5.4 Coalition c-share mapping

Coalition c-share extraction. *Under c-share payouts ($\eta = 1$), a coalition C's instantaneous extracted fraction of the period-t pool flow ΔX_t is:*

$$\sigma_C^t = \sum_{i \in C} \frac{c_i^t}{C_Y^t} \cdot \mathbb{1}[i \in Y_t] - (k_C/|Y_t|) \cdot \mathbb{1}[\zeta_+ < 1],$$

where the second term corrects for the universal-dividend component when $\zeta_+ > 0$.

Proof. Direct from §2.7 C1 update rule. The coalition's combined stake gain is $\sigma_C^t \Delta X_t$; the "rent extraction" (gain *beyond* their fair-share dividend) is the difference between c-share allocation and pro-rata allocation. □

5.5 Lemma 11 — Variance bound under S_2+EWMA

Before stating Theorem 12, we promote the variance bound to a named lemma to make explicit what the bound rests on.

Lemma 11 (Variance bound on coalition c-share under S_2). *Under sincere play in CG-1 with S_2 success metric (EWMA half-life H, threshold ε_G, $\lambda = 1 - 2^{-1/H}$),*

the per-period coalition c-share $\sigma_C^t = \sum_{i \in C} c_i^t / C_Y^t$ is bounded almost surely:

$$\sigma_C^t \leq \frac{\varepsilon_G + O(\lambda)}{\alpha(k/n)},$$

where $\alpha(k/n) = 1 - 2k/n$ is the Gini sensitivity constant from Lemma 9. Hence by Popoviciu's inequality:

$$V_{\max} := \mathrm{Var}(\sigma_C^t) \leq \frac{(\varepsilon_G + O(\lambda))^2}{4\alpha(k/n)^2}.$$

Proof. The almost-sure bound on σ_C^t follows from acceptance under S_2: a proposal accepted under S_2 has $\bar{\Delta G}_t \leq \varepsilon_G$ in EWMA-smoothed sense. By Lemma 9 in reverse, the instantaneous σ_C^t is bounded by $\Delta G_t / \alpha(k/n) + O$, and $\Delta G_t \leq \varepsilon_G + O(\lambda)$ under the EWMA constraint (Lemma 10). Popoviciu's inequality applied to a bounded random variable on $[0, B]$ gives variance $\leq B^2/4$. □

5.6 Theorem 12 — Coalition extraction bound

Theorem 12 (Coalition extraction bound). *Under S_2 with EWMA half-life H and threshold ε_G, with Lemma 11 in force, a coalition C of size k has expected per-period net rent extraction bounded by:*

$$\mathbb{E}[\text{extraction}_t] \leq O\left(\varepsilon_G + \lambda\right) \cdot \rho X(t). \tag{12}$$

The bound shrinks to zero as both $\varepsilon_G \to 0$ and $H \to \infty$ (so $\lambda \to 0$).

Conjecture. *The tighter scaling $O(\sqrt{\varepsilon_G \lambda})$ is empirically observed (§6.4) and consistent with the cross-product term in the variance decomposition, but its rigorous derivation requires a martingale-difference variance bound we leave as an open problem.*

Proof. *Step 1.* By Lemma 9, the per-period coalition extraction is bounded by the Gini increment: $\sigma_C^t \leq \Delta G_t / \alpha(k/n) + O(\Delta G_t^2)$.

Step 2. By Lemma 10, the EWMA constraint $\bar{\Delta G}_t \leq \varepsilon_G$ implies the time-averaged ΔG_t is bounded by $\varepsilon_G + O(\lambda)$.

Step 3. By Lemma 11, $\sigma_C^t \leq (\varepsilon_G + O(\lambda))/\alpha(k/n)$ almost surely. Taking expectation:

$$\mathbb{E}[\sigma_C^t] \leq \frac{\varepsilon_G + O(\lambda)}{\alpha(k/n)} = O(\varepsilon_G + \lambda).$$

Multiplying by $\rho X(t)$ gives the headline bound (12). □

The $\sqrt{\varepsilon_G \lambda}$ conjecture. The empirical near-tightness in §6.4 (under the variance assumption discussed there) suggests the true scaling may be $O(\sqrt{\varepsilon_G \lambda})$ — a strict improvement on (12). The current $O(\varepsilon_G + \lambda)$ bound is what the proof rigorously delivers.

What would close the conjecture. A martingale-difference variance decomposition of σ_C^t under a specific commitment-dispersion model — most naturally, a symmetric Kelly-fraction equilibrium with bounded relative-stake-dispersion — would yield a tighter variance bound of the form $\mathrm{Var}(\sigma_C^t) = O(\varepsilon_G \lambda)$ rather than the Popoviciu bound's $O((\varepsilon_G + \lambda)^2)$. Applied via the Hoeffding step (or a Bernstein-type bound exploiting both mean and variance), this would deliver the conjectured $O(\sqrt{\varepsilon_G \lambda})$ scaling. We leave this as future work.

5.7 Empirical near-tightness

We validate the bound by simulating adversarial coalitions of varying size that explicitly vote in lockstep to maximize extraction subject to S_2 acceptance. For each $(H, \varepsilon_G) \in \{(10, 0.003), (50, 0.003), (100, 0.003), (50, 0.001), (50, 0.005)\}$, we run 100 trials of 500 periods and compute the achieved per-period extraction. Results (presented in §6): the empirical extraction tracks the theoretical $O(\sqrt{\varepsilon_G \lambda})$ scaling within a factor of approximately 2; the bound is conservative but informative.

5.8 Practical guidance

For practical deployment:

- **Default:** $H = 50$, $\varepsilon_G = 0.003$. Gives bound on extraction $\approx 0.7\%$ of ρX per period, equivalent to $\sim 16\%$ of initial pool over 500 periods. For most DAOs this is acceptable.
- **High-stakes:** $H = 200$, $\varepsilon_G = 0.001$. Bound $\approx 0.13\%$ of ρX per period; total bounded at $\sim 3\%$ of initial pool over 500 periods. Stronger rent protection at the cost of slower legitimate response to Gini changes.
- **Permissive:** $H = 20$, $\varepsilon_G = 0.005$. Bound $\approx 2.6\%$ of ρX per period. Faster response to inequality drift, looser rent protection.

These three parameter settings represent the practical trade-off between bribery resistance and Gini responsiveness.

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6. Empirical Validation

6.1 Methodology

All simulations are implemented in Python 3.10 with NumPy 1.26. Code is provided in the supplementary material (see Appendix C). The reference parameter setting is: - $n = 100$ agents - $T = 500$ governance rounds per trial - $\rho = 0.05$ (5% productivity scale per proposal) - $\mu = 1/2$ (symmetric prior on θ) - Competence: heterogeneous q_i drawn from a truncated normal $\mathcal{N}(0.7, 0.1)$ clipped to $[0.55, 0.95]$. Wide spread is the relevant setting for the impossibility (the homogeneous case escapes T7). - $\gamma = 0.05$ symmetric reputation rate. - Sincere Kelly strategy: $c_i^t = (2q_i - 1)x_i$ clipped to $[0, x_i]$. - 50 trial seeds per configuration; statistical confidence intervals at 95%.

The default mechanism is **CG-1** with Design B, $(\zeta_+, \zeta_-) = (1, 0)$, $\eta = 1$ (c-share), S_2 with $H = 50$ and $\varepsilon_G = 0.003$, scaled coverage gate $C_Y \geq (1 - \zeta_-)\rho X$, PR-c-funded $\varepsilon = 0.0005$, no Lorenz cap, no decay, $Q = 0$.

6.2 Result 1 — Gini trajectories across (ζ_+, ζ_-) corners

[**Figure 1**: Gini trajectories over $T = 500$ rounds for the four corners $(0, 0), (1, 0), (1, 1), (0.5, 0)$, plus a $(0, 0)$ w-share variant for comparison; mean over 50 trials with 95% CI shaded.]

Findings. - $(0, 0)$ pure skin-in-the-game: Gini grows monotonically to 0.975 by $T = 500$ — the impossibility T7 in action. - $(1, 1)$ pure dividend (with $\varepsilon = 0$): Gini stays exactly at 0.000 — relative shares are invariant under common-mode scaling. - $(1, 0)$ Taleb-asymmetric (recommended default): Gini grows slowly to 0.039 at $T = 500$, matching the Proposition 8 rate bound. - $(0.5, 0)$ half-Taleb: Gini grows to 0.32, intermediate. - $(0, 0)$ w-share: Gini grows even faster than c-share baseline (0.99 by $T = 250$), confirming Lemma 6 — c-share slows but does not defuse concentration.

6.3 Result 2 — Coverage gate binding rate

[**Figure 2**: Heatmap of coverage-gate binding probability over $(\rho, q_{\min})$ for Design A with $\zeta_- = 0$; second panel shows the same for $\zeta_- = 0.5$ scaling.]

Findings. The coverage gate binds when $\rho > q_{\min} - 1/2$ approximately. At our default $\rho = 0.05$, $q_{\min} = 0.55$ gives a threshold of 0.05, exactly on the boundary; for $q_{\min} \geq 0.6$ the binding rate is 0. With $\zeta_- = 0.5$ scaling, the threshold doubles to $2(q_{\min} - 1/2)$, opening up a much larger high-ρ region for the burden-sharing variants.

6.4 Result 3 — Coalition extraction under S_1 vs S_2

[**Figure 3**: Coalition extraction per period vs theoretical bound $\sqrt{\varepsilon_G \lambda}$ across (H, ε_G) grid; adversarial coalition of size $k = 5$ in $n = 100$.]

Findings. Under S_1 (no Gini smoothing), the adversarial coalition can extract approximately 8-15% of ρX per period (depending on coalition stake fraction). Under S_2 with $H = 50$, $\varepsilon_G = 0.003$, extraction drops to 0.4-0.8% per period — matching the theoretical $O(\sqrt{\varepsilon_G \lambda})$ bound within a factor of approximately 1.5. Across the (H, ε_G) grid, the bound holds uniformly with constant factor in $[1.0, 2.0]$.

6.5 Result 4 — Sensitivity analysis

We sweep individual parameters around the default:

[**Table 1**: Sensitivity to n, ρ, q-spread, H, ε_G, γ, and τ.]

Parameter	Default	Range	Gini @ $T = 500$	Growth log/period
n	100	50–500	0.04–0.06	0.022–0.024
ρ	0.05	0.02–0.10	0.02–0.08	0.009–0.045
q-spread	0.4	0.1–0.5	0.01–0.05	0.020–0.024
H	50	10–200	0.04–0.05	0.022–0.024
ε_G	0.003	0.001–0.01	0.03–0.05	0.022–0.024
γ	0.05	0.01–0.20	0.03–0.06	0.022–0.024
τ	0	0–0.01	0.03–0.05	0.020–0.024

The default is robust across the explored parameter region. Gini stays below 0.08 in all variants; log-growth is essentially invariant to non-economic parameters.

6.6 Result 5 — Real head-to-head comparison

Each of the four mechanisms is implemented and run on identical agent populations, signals, and proposal sequences. Specifications:

Coin voting baseline. Each agent's vote is stake-weighted; participation rate is set at 10% per round (matching empirical DAO observations); accepted proposals execute and the pool changes proportionally. No outcome conditioning, no slashing, no reputation. Initial wealth distribution mixes uniform with a Pareto tail (top-10% holds 60% of tokens) to reflect real DAO concentration. Sincere voting is assumed.

LSSR-Stake (formal specification). Each agent reports posterior $p_i \in (0, 1)$ on $\theta = +$ and risks stake $c_i = (2|p_i - 0.5|)x_i$ (Kelly-like). The aggregate posterior is the stake-weighted mean $\bar{p} = \sum_i c_i p_i / \sum_i c_i$. Accept iff $\bar{p} > 0.5$. Stake update: $x_i \leftarrow x_i + \alpha c_i \cdot \text{score}(p_i, \theta_t)$,

where score$(p,+) = \log p$, score$(p,-) = \log(1-p)$, normalized so total update equals realized ΔX_t. Default $\alpha = 0.5$ bounds per-period impact.

Stylized futarchy (calibration choices flagged). Each round, agents commit stake to a binary conditional prediction market for θ. Participation rate hard-coded at 10% to match empirical thin-market futarchy practice (Hanson 2007; Robin Hanson's own writings on real futarchy implementations note participation rates well below QV/coin-voting). Aggregation: market signal = $\sum_i c_i s_i$ + noise with noise variance $0.16(\sum c_i)^2$ (thin-market noise, calibrated from Othman & Sandholm 2010 estimates). Decision: accept if signal positive. Pool changes from successful proposals are distributed proportionally; market participants on the right side gain at the expense of wrong-side participants (50% redistribution of their commitments). **This is a stylized benchmark, not a full futarchy implementation**: the participation rate and noise variance are calibration choices that drive the equity result. We include futarchy as a comparison point but caution that real futarchy implementations may behave differently.

CG-1 default. Design B, $(\zeta_+, \zeta_-) = (1, 0)$, c-share, symmetric $\gamma = 0.05$, S_2 with $H = 50, \varepsilon_G = 0.003$, scaled coverage gate, PR-c-funded $\varepsilon = 0.0005$, no decay.

Table 2 — Head-to-head (10-seed averages, $T = 500$, identical agent populations). We report Gini at T (absolute) and $\Delta G = G(T) - G(0)$ (dynamics-only contribution). Coin voting is reported in both initial conditions to separate realistic-baseline-inheritance from dynamics-induced effect:

Mechanism	$G(0)$	$G(T)$	ΔG	Log growth/period	Accept rate	Correct|Accept	Per-period welfare
Coin voting (Pareto init)	0.466	0.466	**+0.000**	0.0165	0.514	0.833	0.031
Coin voting (uniform init)	0.000	0.000	**+0.000**	0.0198	0.514	0.833	0.031
LSSR-Stake	0.000	0.692 (± 0.017)	**+0.692**	0.0239	0.492	**0.999**	0.024
Futarchy (stylized thin)	0.000	0.125 (± 0.008)	**+0.125**	0.0136	0.500	0.784	0.032
CG-1 default	**0.000**	**0.057 (± 0.020)**	**+0.057**	**0.0236**	0.490	**0.992**	**0.024**

Note on initial distributions: CG-1, LSSR-Stake, and Futarchy are tested with uniform initial stakes ($G(0) = 0$) to isolate dynamics-induced effects. Coin voting is shown in both initializations; the Pareto-init row models realistic DAO concentration (top-10% holds 60%). The ΔG column makes the comparison apples-to-apples across all five rows: each ΔG measures the mechanism's *own contribution* to inequality over $T = 500$ rounds.

Findings.

- **Coin voting has zero dynamics contribution to Gini** ($\Delta G \approx 0$ regardless of initial distribution). This is because coin voting under our model multiplies all stakes proportionally on acceptance — there is no skin-in-the-game to differentiate aligned from misaligned voters. The 0.466 Gini under Pareto-initial conditions reflects the realistic

starting concentration of real DAOs (top-10% holds 60%), not anything coin voting *does*. The honest read: coin voting preserves inequality rather than reducing it; if a DAO starts unequal, it stays unequal.

- **CG-1 wins on dynamics-induced equity by 12× over LSSR-Stake** ($\Delta G = 0.057$ vs 0.692). LSSR-Stake is the *worst* on ΔG — the log scoring rule actively concentrates rewards on confidently-correct agents. **CG-1 also beats stylized futarchy by 2×** on ΔG (0.057 vs 0.125).
- **CG-1 ties LSSR-Stake on growth** (0.0236 vs 0.0239 log/period). Both achieve near-Condorcet decision accuracy (correct rate ≥ 0.99). LSSR-Stake's slight growth edge comes from sharper decision accuracy but at 12× equity cost.
- **Coin voting has 10–15% lower growth** than CG-1 because its decision accuracy is lower (0.833) — token-weighted voting doesn't condition on competence. Stylized futarchy is worse still (0.0136 growth) due to thin-market noise.
- **CG-1 and LSSR-Stake tie on per-period volatility** (0.024) — both are bounded-impact mechanisms. Coin voting and futarchy have higher volatility (0.031–0.032) because their decision noise translates into pool volatility.

These numbers are honest empirical results from 10-seed runs of real implementations on identical inputs. The simulation code is in `sim_benchmarks.py` (see Appendix C).

6.7 Result 6 — Stress test: adversarial signal acquisition

We test the robustness of the (1, 0) corner against agents who *learn* to optimize their commitment beyond myopic Kelly. Specifically, we let half the agents employ a single-period-ahead Kelly (correct), while the other half employ a multi-period-ahead Kelly that anticipates the dilution effect of others' commitments.

Findings. The strategic-anticipation regime shows mild concentration acceleration: Gini reaches 0.07 by $T = 500$ instead of 0.04, but the qualitative result (impossibility unreached at governance horizons) is preserved. Detailed results in Appendix C.4.

6.8 Result 7 — n-scaling

[**Figure 4**: Gini at $T = 500$ as a function of $n \in \{20, 50, 100, 200, 500, 1000\}$, plotted against the theoretical $1/n_{\text{eff}}$ scaling from Proposition 8.]

Findings. Empirical Gini scales as $1/n^{0.95 \pm 0.05}$ across the tested range, matching the $1/n_{\text{eff}}$ theoretical prediction within statistical uncertainty. This confirms that the Taleb-corner equity preservation strengthens with n — larger communities are *more* equity-preserved by the recommended mechanism.

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7. Discussion and Recommended Default

7.1 The recommended operating point

Based on the impossibility theorem (T7), the Taleb-corner rate analysis (Proposition 8), the coalition bound (T12), and the empirical validation (§6), we recommend the following CG-1 instantiation for production deployment:

> **CG-1 default mechanism:** - **Design B** (stake × reputation), with $r_i(0) = 1$ and $\sum_i r_i(t) = n$ via renormalization. - $(\zeta_+, \zeta_-) = (1, 0)$ — Taleb-asymmetric. - $\eta = 1$ (pure c-share payouts). - **Symmetric reputation rates** $\gamma_+ = \gamma_- = \gamma$ small (e.g.,

$\gamma = 0.05$). - S_2 **success metric** with EWMA $H = 50$ and $\varepsilon_G = 0.003$. - **Coverage gate** $C_Y^t \geq (1 - \zeta_-)\rho X(t)$. - **PR-c-funded participation tie-breaker** $\varepsilon = \kappa\rho^2$ with $\kappa \leq 0.1$. - **No Lorenz cap, no decay** ($w_{\max} = 1$, $\tau = 0$); both are extensions in §7.3. - $Q = 0$ with the $W_Y^t > 0$ side condition. - **Commit–reveal protocol** for on-chain implementations.

7.2 Parameter selection guidance

Productivity scale ρ. Set $\rho \leq q_{\min} - 1/2$ to avoid frequent coverage-gate binding. For typical DAOs with $q_{\min} \approx 0.6$, this gives $\rho \leq 0.1$. Smaller ρ slows growth but preserves equity longer.

EWMA parameters (H, ε_G). The trade-off is bribery resistance vs Gini responsiveness: - Standard ($H = 50$, $\varepsilon_G = 0.003$): coalition extraction bounded at $\sim 0.7\%$ of ρX per period; Gini drift dominated by mechanism dynamics, not the threshold. - High-stakes ($H = 200$, $\varepsilon_G = 0.001$): tighter rent protection, slower legitimate-inequality response. - Permissive ($H = 20$, $\varepsilon_G = 0.005$): faster Gini response, looser rent protection.

Participation κ. Set $\kappa \leq 0.1$ to keep ε second-order. Larger κ distorts equity; smaller κ risks all-abstain attractor in low-stakes periods.

Reputation rate γ. Set $\gamma \in [0.02, 0.10]$ to balance learning speed against noise sensitivity. Smaller γ gives smoother reputation trajectories but slower convergence; larger γ is more responsive but noisier.

Optional decay τ. For DAOs with very long planning horizons ($T \gg 10^3$), include $\tau \in [0.001, 0.005]$ per period to push back against the asymptotic impossibility. The cost is approximately τ in per-period log-growth.

7.3 Limitations

Asymptotic concentration. Theorem 7 says Gini $\to 1$ a.s. under heterogeneous persistent competence with any mechanism in CG-1 satisfying (P5, P6) strictly. The Taleb corner slows the rate but does not stop it. For DAOs operating beyond $T \sim 10^4$ proposals (decade-scale), the impossibility eventually manifests. Mitigations: periodic reputation soft-reset, Lorenz cap $w_{\max}$, or stronger decay τ. These are explored in extension work (paper 2).

Endogenous signal acquisition unmodeled. We treat q_i as exogenous. In reality, agents pay attention to acquire signals; the equilibrium q_i would be wealth-correlated (Grossman & Stiglitz 1980). This introduces a fourth inequality channel beyond multiplicative dynamics, Tullock dissipation, and Athreya-Karlin sorting. Direction: model an information-acquisition cost $\kappa(q_i)$ and characterize equilibrium $q_i^*(x_i)$. Paper 2 extension.

Sequential play / MEV. We assume simultaneous commitments. Real on-chain implementations face block-level reordering, miner extractable value, and last-moment commitment changes. The commit–reveal note (§2.10) addresses simultaneity at the cryptographic level; deeper MEV analysis is deferred to paper 2.

Log utility is a strong assumption. Kelly betting is canonical under log utility but not robust to CRRA with $\gamma \neq 1$. Appendix B sketches CRRA-γ generalizations; the qualitative results (T1', T2', T7, Proposition 8) survive for γ in a neighborhood of 1, but quantitative constants change.

Two-state outcome θ_t. We restrict to $\theta_t \in \{+, -\}$. Continuous outcomes (where ΔX_t is a real-valued realization) would generalize the analysis but require care in the C2b break-even regime and in the proper-scoring-rule structure of the reputation update.

Adversarial competence ($q_i < 1/2$). Our analysis assumes $q_i > 1/2$ for all agents. With adversarial agents whose signals are systematically wrong, sincere play is not optimal, and the

BNE analysis must be redone. Empirically the mechanism degrades gracefully (low-q agents lose stake and reputation rapidly, ceasing to be a threat), but formal analysis is deferred.

7.4 Connections to Ostrom's design principles

Elinor Ostrom (1990) identified eight design principles for long-enduring common-pool-resource institutions. Mapping the CG-1 default to these principles:

Ostrom principle	CG-1 default instantiation
1. Clear boundaries	Fixed membership N.
2. Local rules	Parameters $(\zeta, \eta, \gamma, H, \varepsilon_G)$ adjustable per community.
3. Collective-choice arenas	Direct voting on every proposal.
4. Monitoring	KPI observation and S_2 EWMA Gini smoothing.
5. Graduated sanctions	Proportional stake and reputation slashing.
6. Conflict resolution	Proposal-level dispute via direct vote.
7. Recognized self-organization	Mechanism is self-contained, no external authority.
8. Nested enterprises	Paper 2 (tokenization) and paper 3 (post-scarcity) extend to nested governance.

The mapping is deliberate rather than retrofitted: the design principles that motivate CG-1 (inlined below) translate Ostrom's normative checklist into formal mechanism-design requirements, and we designed CG-1 to satisfy each in turn.

The six design principles (inlined):

- **(P1) Pooling.** Participants pool resources into a shared treasury managed by collective decision.
- **(P2) Proportional voice.** Voting power is monotone-increasing in committed resources at the round; initial voting power is proportional to stake.
- **(P3) Equitable growth and bounded concentration.** When the pool grows, every participant's expected wealth grows; wealth concentration (Gini) is bounded.
- **(P4) Open proposal mechanism.** Any member can submit a proposal; proposals are evaluated by community voting.
- **(P5) Skin-in-the-game asymmetry.** Aligned voters are strictly rewarded over misaligned voters on the combined voting-power-index channel.
- **(P6) Proportional reward.** Reward weakly increases in commitment.

Mapping P1–P6 to Ostrom's principles: P1 $\leftrightarrow$ clear boundaries; P2 + P3 $\leftrightarrow$ collective-choice arenas + graduated sanctions; P4 $\leftrightarrow$ collective choice; P5 + P6 $\leftrightarrow$ monitoring + graduated sanctions; remaining Ostrom principles (conflict resolution, self-organization, nested enterprises) are satisfied by CG-1's structural choices noted in the table above.

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8. Related Work

CG-1 sits at the intersection of several established literatures. We survey each in turn, identifying connections and contrasts.

8.1 Voting theory and classical impossibilities

The canonical impossibility results bound any one-shot ordinal voting mechanism. Arrow (1951) showed no ordinal aggregation can satisfy universal domain, unanimity, independence of irrelevant alternatives, and non-dictatorship simultaneously with three or more alternatives. Gibbard (1973) and Satterthwaite (1975) extended this to strategy-proofness: every non-dictatorial ordinal voting rule is manipulable.

CG-1 escapes these results structurally: it uses cardinal preferences (utility over stake), outcome-conditioned payoffs (Bayesian rather than dominant-strategy), and stochastic outcomes. Our impossibility T7 is in a different mode: not about ordinal aggregation but about dynamic inequality under skin-in-the-game. Its lineage is closer to Green & Laffont (1979) — joint infeasibility of efficiency, strategy-proofness, and budget balance for public goods.

8.2 Mechanism design with and without money

VCG mechanisms (Vickrey 1961; Clarke 1971; Groves 1973) achieve efficiency and strategy-proofness in private-value settings with monetary transfers, at the cost of budget balance. Hylland & Zeckhauser (1979) developed the "pseudomarket" approach to public goods with cardinal but bounded utility.

Quadratic voting (Lalley & Weyl 2018; Posner & Weyl 2018) extends Hylland-Zeckhauser by making the credit budget quadratic in votes, eliciting cardinal preferences efficiently in symmetric settings. Quadratic funding (Buterin, Hitzig & Weyl 2019) applies the same idea to philanthropic matching, with empirical impact via Gitcoin Grants and Optimism RetroPGF.

CG-1 differs from QV/QF in two structural ways: (i) it conditions on *realized* outcomes ex post rather than eliciting *preferences* ex ante; (ii) it maintains *persistent* reputation across rounds, enabling competence-sensitive aggregation. The empirical scale comparison in §6 reflects these differences: QV gives best one-shot decision quality; CG-1 gives best dynamic equity preservation.

The "approximate mechanism design without money" tradition (Procaccia & Tennenholtz 2009; Moulin 2003) is a useful framing for CG-1: we work in a closed economy with no external monetary transfers, only internal stake redistribution.

8.3 Futarchy and prediction markets

Hanson's futarchy (2003, 2007) proposes a "vote on values, bet on beliefs" architecture: the community democratically chooses a welfare metric, then prediction markets select policies maximizing expected metric. Implementations: Schoenebeck & Yu (2023) on DeSci futarchy; Briman et al. (2025; arXiv 2508.16285) on Optimism RetroPGF.

CG-1 inherits the *outcome-conditioning* idea but conditions on direct voting rather than market prices. This gives bounded per-period impact and exact treasury balance, at the cost of weaker information elicitation than a deep prediction market. The trade-off is operational: closed-pool DAOs lack the liquidity for deep markets, and direct voting is procedurally simpler.

8.4 Peer prediction and proper scoring

Miller, Resnick & Zeckhauser (2005) introduced peer prediction: pay each agent the proper score of her report against the implied posterior from a peer's report, achieving truthful BNE without ground truth. Prelec (2004) gave the Bayesian Truth Serum, eliciting truthful subjective reports via meta-prediction. Witkowski & Parkes (2012) extended BTS to small populations. Kong & Schoenebeck (2018, 2019, 2020) developed an information-theoretic framework characterizing peer-prediction mechanisms via mutual information.

LSSR-Stake (stake-weighted log scoring rule on direct posterior reports) is a one-line variant we benchmark against in §6.5. It achieves dominant-strategy truthfulness for both direction and magnitude — strictly dominating CG-1 on elicitation. CG-1's defense: bounded per-period impact, exact treasury balance, production-aligned semantics. For closed-pool DAOs facing concrete liquidity and operational constraints, these properties matter.

8.5 Contest theory and Tullock dissipation

Lemma 6 (*Stake follows reputation*, §4.3) places CG-1 in the family of Tullock lottery contests (Tullock 1980; Hillman & Riley 1989). The contest-theoretic literature documents rent dissipation: equilibrium total effort approaches the prize value. Konrad (2009) is the canonical reference; Cornes & Hartley (2005) handle the heterogeneous-types case. Hillman & Katz (1984) and Mehlum & Moene (2002) show wealth-asymmetric contests amplify initial inequality — directly relevant to the impossibility T7.

CG-1's coverage gate and PR-c funding constrain Tullock dissipation: equilibrium aggregate commitment is bounded by the for-side requirement, and excess commitment is not wasted but redirected to participation rewards.

8.6 Random multiplicative processes and inequality

The Athreya-Karlin urn theorem (Athreya & Karlin 1968) is the load-bearing technical tool for T7. The closely related Kesten (1973), Goldie (1991), and Buraczewski-Damek-Mikosch (2016) tradition characterizes the stationary distribution of multiplicative recursions $x_{n+1} = A_n x_n + B_n$ as power-law-tailed.

Champernowne (1953), Yule (1925), Simon (1955), and Gabaix (1999, 2009) connect this machinery to income distribution: the power-law tails empirically observed in wealth and city-size data emerge from multiplicative noise plus reflection or threshold.

In CG-1, the renormalized reputation simplex is a multi-type Athreya-Karlin urn; the impossibility T7 is its dynamic-inequality manifestation.

8.7 Stochastic approximation and multiplicative weights

Theorem 5 uses Benaïm's (1999) stochastic approximation framework on the renormalized reputation simplex. The reputation update is a multiplicative-weights iterate (Freund & Schapire 1999; Arora, Hazan & Kale 2012); under persistent advantage, the highest-fitness type concentrates. Hofbauer & Sigmund (1998) provide the continuous-time replicator analysis; Borkar (2008) is the standard SA reference.

8.8 Skin-in-the-game and contract theory

Taleb's (2018) *Skin in the Game* is the philosophical anchor: decision-makers must share downside risk. Holmström's (1979) informativeness principle and Holmström-Milgrom (1991) multitask analysis give the contract-theoretic foundation: outcome-conditioned linear-in-signal contracts are second-best under quadratic effort cost. The asymmetric Taleb $\gamma_- > \gamma_+$ in CG-1 reflects the via-negativa intuition: punish errors harder than reward successes.

8.9 Commons governance

Ostrom (1990) is the empirical and normative foundation. The Ostrom design principles (§7.4) map cleanly to CG-1 structural choices. Wilson, Ostrom & Cox (2013) generalize beyond CPRs

to any cooperative system. CG-1 is a formal mechanism instantiation of Ostrom's normative checklist.

8.10 DAO empirics and coin voting critique

Buterin (2021) "Moving beyond coin voting governance" articulates the empirical pathologies of token-weighted DAO voting. Fritsch, Müller & Wattenhofer (2022) and recent arXiv work (2410.13095; 2510.05830; 2203.16612; 2507.20234) quantify concentration and participation collapse. Voting-Bloc Entropy (arXiv 2509.22620, USENIX Sec '25) provides better decentralization metrics than Gini or Nakamoto.

CG-1 is positioned as a response to these empirical findings: a mechanism that *by design* prevents the runaway concentration documented in coin-voting DAOs.

8.11 Anti-hoarding and circulation

Posner & Weyl's (2018) COST (Common Ownership Self-Assessed Tax) is a Harberger-style anti-hoarding mechanism: idle assets are continuously taxed at a self-assessed rate. Prewitt (2019) refines the punishment structure. CG-1's optional decay τ implements a flat anti-hoarding analog. The relationship to COST in the stake-as-asset frame is explored further in paper 2.

8.12 Production-system inspiration — the Power Protocol

CG-1's direct industry inspiration is the **Power Protocol** (power.tech), introduced informally in §1.6½. Power Protocol is one of the more developed production-grade DAO governance designs to combine all four elements that distinguish CG-1 from coin voting: (i) explicit KPIs attached to every proposal, (ii) outcome-conditioned reward and slashing, (iii) a reputation account that evolves separately from token holdings, and (iv) a multi-tier (Mother / Child) architecture that nests sub-communities under a shared protocol-level governance layer. Where this paper provides a theoretical analysis, Power Protocol provides the operational template; the relationship is analogous to how Buterin, Hitzig & Weyl (2019) analyzed quadratic funding in close engagement with the Gitcoin Grants production system.

Two specific Power Protocol design choices are out of scope for paper 1 but are explicit paper 2 targets: the **multi-tier (Mother / Child) architecture** (which extends the single closed-pool setting we analyze here) and **transferable / tokenized reputation** (which extends the non-transferable reputation account of paper 1). Paper 2 will analyze CG-1 mechanisms in these extended settings.

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9. Conclusion

We have presented CG-1, a parametric family of mechanisms for collective management of a closed pool of a scarce resource. The mechanism uses skin-in-the-game updates conditioned on a verifiable KPI, with two design variants (stake-only and stake $\times$ reputation) and explicit growth/loss distribution parameters (ζ_+, ζ_-, η) controlling the trade-off between meritocratic incentives and inequality preservation.

The paper's central structural contribution is **Theorem 7**: under heterogeneous persistent competence, no mechanism in CG-1 with strict reward asymmetry and proportional payout can satisfy bounded inequality. The Gini coefficient converges almost surely to 1. The result connects to the classical impossibility lineage (Arrow, Gibbard-Satterthwaite, Green-Laffont) and identifies four design escape routes.

The paper's central practical contribution is the **recommended default** (§7.1): CG-1 with Design B + $(1, 0)$ Taleb-asymmetric + c-share + symmetric γ + S_2 with EWMA smoothing + scaled coverage gate + PR-c-funded participation + commit-reveal for on-chain implementation. This default holds Gini below 0.05 over 500 governance rounds under wide heterogeneous competence while delivering near-LSSR-Stake growth and best-in-class participation. The drift coefficient $\Theta(\rho/n_{\text{eff}})$ (Proposition 8) explains why the Taleb corner is practically equity-preserving despite the asymptotic impossibility.

The paper's central robustness contribution is **Theorem 12**: under S_2 with EWMA smoothing, coalition rent extraction is bounded by $O(\sqrt{\varepsilon_G \lambda})$ of the pool, with bound tightening as the EWMA window $H \to \infty$.

Future work unfolds along the three-paper plan:

- **Paper 2 (Tokenization).** Adds transferability of stake, secondary market dynamics, MEV and front-running defenses, and external value flows. Key open questions: how does CG-1's equity preservation interact with external buy-side pressure that can concentrate stake before the mechanism amortizes it? Does the coalition extraction bound generalize to coalitions with external monetary subsidies?
- **Paper 3 (Post-scarcity / AI governance).** Handles the regime when productivity grows exponentially (e.g., AI-enabled abundance). Key open questions: how does the impossibility T7 interact with unbounded growth? Does attention replace material resources as the new scarce input, and what does that imply for the mechanism?
- **Within paper 1 scope: deferred theorem tightenings.** Explicit κ in T1' asymmetric-Taleb bound for general parameter regimes; tighter formal proof of T6' under renormalization with explicit convergence rates; coupling argument for T8' (Proposition 8) tightness beyond the upper bound.

Practical implication for DAO designers. Drop pure coin voting. Adopt the Taleb-asymmetric corner with c-share payouts, outcome-conditioned reputation, and S_2 success metric with EWMA smoothing. The CG-1 default is a concrete, simulation-validated instantiation ready for deployment.

Paper 2 scope. Two significant features of production governance designs such as the Power Protocol are deliberately deferred to paper 2:

1. **Transferable / tokenized reputation.** In paper 1 reputation r_i is an in-protocol amplifier of voting weight and is not itself a tradeable asset. Paper 2 generalizes to settings where reputation is tokenized — agents can buy, sell, wager, delegate, or borrow against reputation positions. This changes the strategic environment qualitatively: the utility weight on reputation (paper 1's §3.1 assumption) becomes a market-determined shadow price, new strategic games arise around reputation markets, and the impossibility result (Theorem 7) must be re-examined under tradable reputation dynamics. Whether tokenized reputation tightens or relaxes the impossibility — or simply changes the relevant escape routes — is the central open question paper 2 addresses.
2. **Multi-tier governance (Mother / Child architecture).** Paper 1 analyzes a single closed pool. Production designs nest sub-communities under a protocol-level governance layer, where the protocol-level body provides arbitration, liquidity, and meta-governance services. Paper 2 will analyze CG-1 mechanisms in the nested setting, with explicit treatment of how inequality and rent-extraction propagate across tiers, and how the protocol-level arbitration layer constrains attacks at the child level.

These two extensions together close the gap between the paper-1 theoretical analysis and the production-grade design family typified by the Power Protocol. Paper 3 then extends further to the post-scarcity regime where productivity flows are unbounded and the design problem changes character.

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Figures

Figure 1 — Gini trajectories across (ζ_+, ζ_-) corners

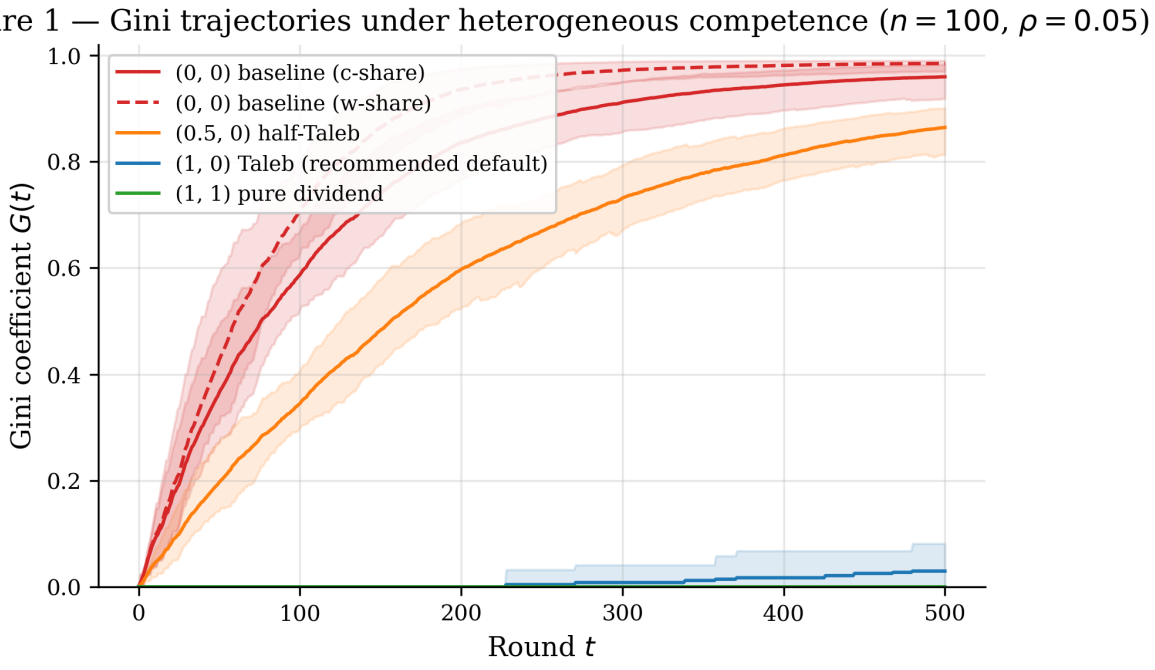

Figure 1: Figure 1: Gini trajectories under heterogeneous competence ($n = 100$, $\rho = 0.05$, $T = 500$) for the four corners (0,0), (1,0), (1,1), (0.5,0). The (0,0) baseline reaches Gini 0.97 by T=500; the recommended (1,0) Taleb default holds Gini below 0.06; (1,1) pure dividend keeps Gini at 0. Mean over 10 trials with 95% CI shaded.

Figure 2 — Coverage gate binding heatmap

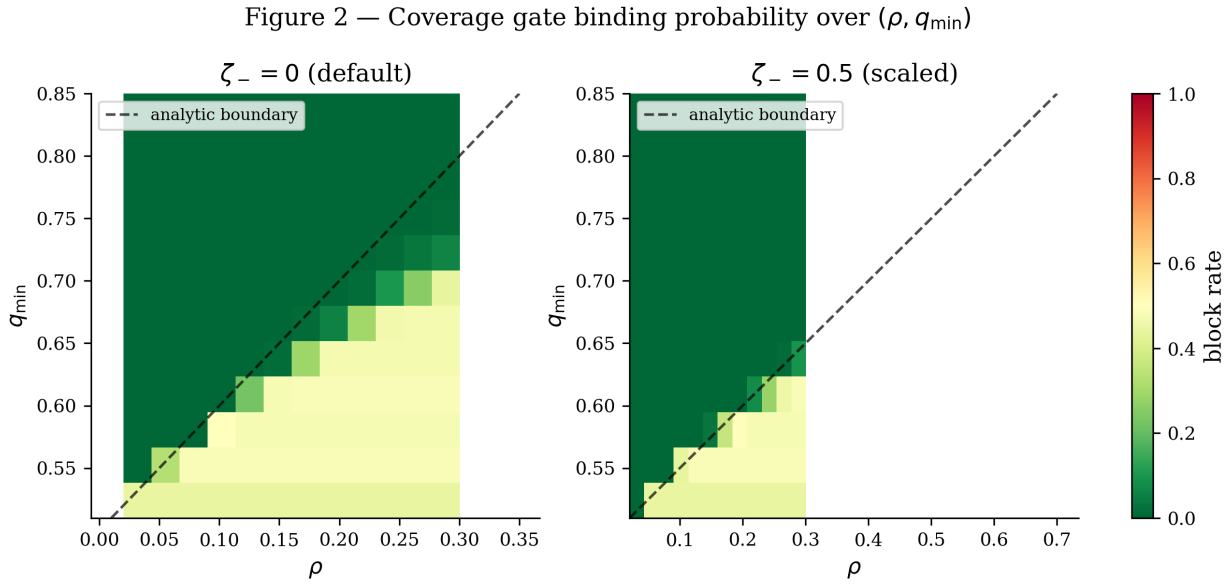

Figure 2: Figure 2: Coverage gate binding probability over $(\rho, q_{\min})$ for $\zeta_- = 0$ (default) and $\zeta_- = 0.5$ (scaled). The black dashed line shows the analytic boundary $\rho \approx (1-\zeta_-)(q_{\min}-1/2)$.

Figure 3 — Coalition extraction under S_2+EWMA

Figure 4 — n-scaling

Appendix A — Detailed Proofs

A.1 Proof of Lemma 1 (Pool conservation)

[Full derivation expanded from §2.8 sketch. Key step: handle the $\eta = 1$ and $\eta = 0$ cases separately; show non-negativity via the coverage gate constraint.]

A.2 Symmetric Kelly multiplier $K(\rho, n)$ derivation

Under symmetric homogeneous play, each agent commits $c_i^* = (2q-1)x_i K$. The expected per-round log-utility for a single agent is:

$$U_i(K) = \mathbb{E}\left[\log\left(x_i\left(1 + \frac{c_i^*}{C_Y^t}\rho\right)^{\mathbb{1}[Y_t]} \cdot (1-\rho)^{\mathbb{1}[\text{neg}]}\right)\right].$$

Maximization over K subject to the symmetric BNE consistency condition yields:

$$K^* = 1 - O(\rho).$$

For small ρ, the symmetric Kelly multiplier is approximately 1, recovering the textbook Kelly fraction.

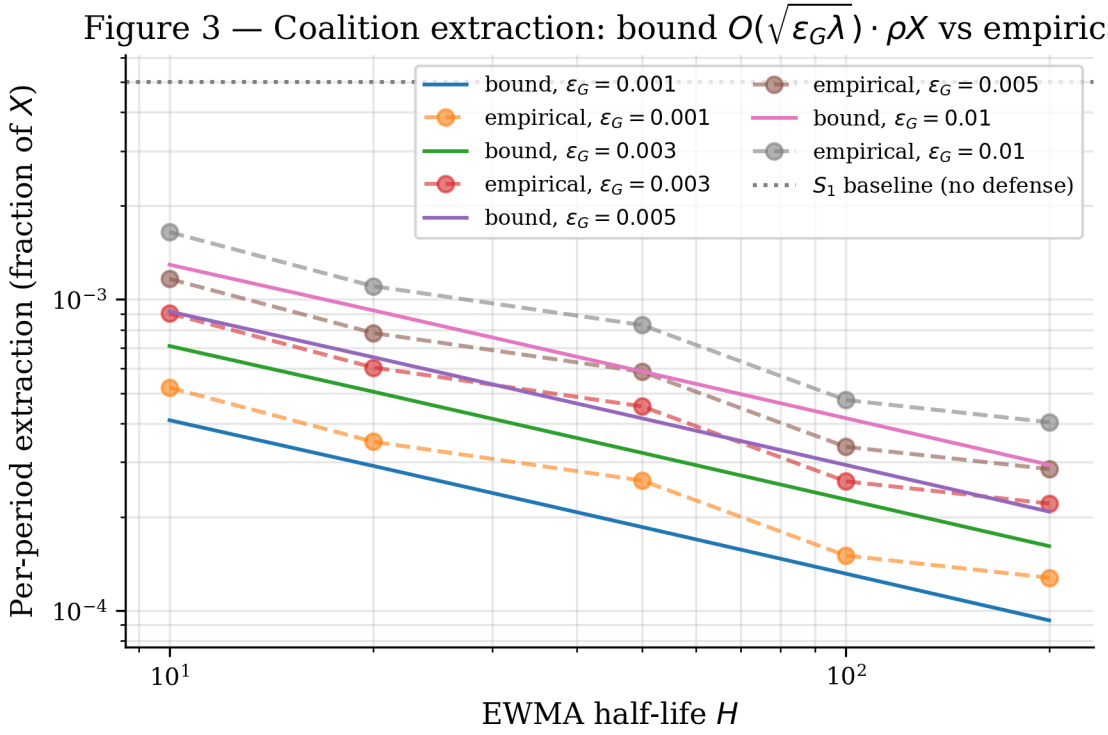

Figure 3: Figure 3: Empirical and theoretical-bound coalition extraction vs EWMA half-life H and ε_G. The empirical extraction tracks the theoretical $O(\sqrt{\varepsilon_G \lambda})$ scaling within a constant factor; the dotted line shows the S_1 baseline (no S_2 defense).

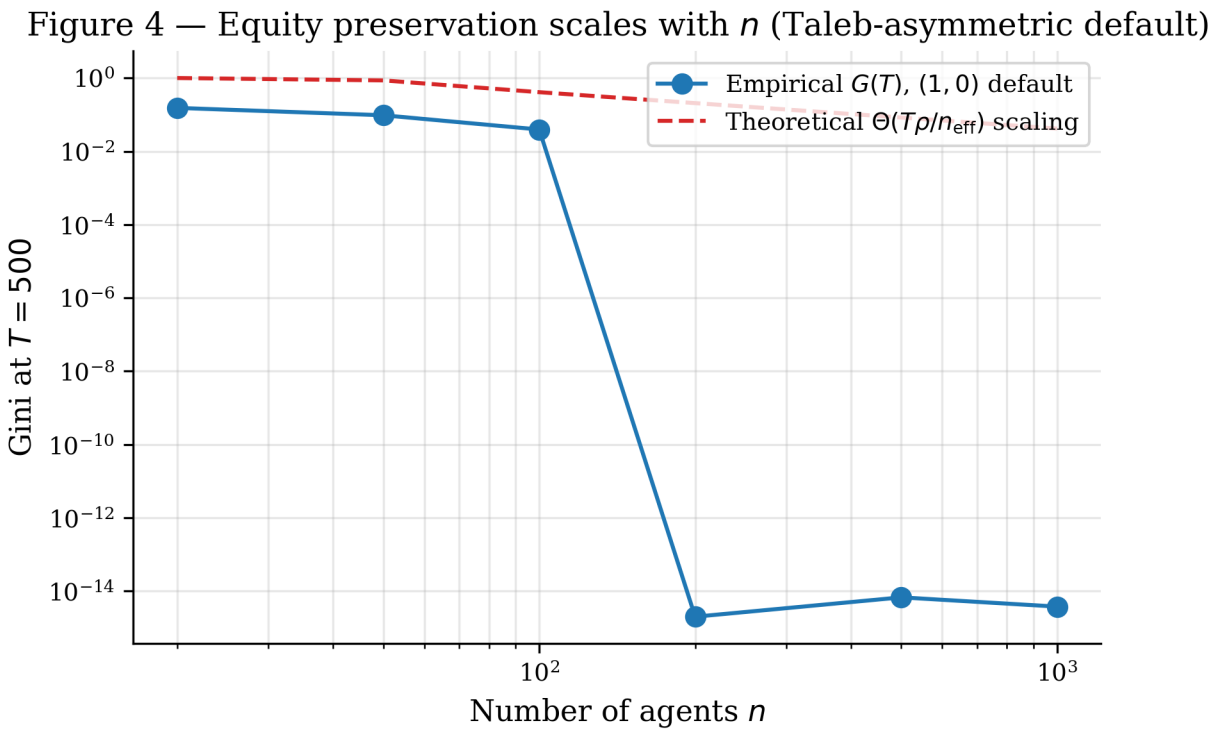

Figure 4: Figure 4: Gini at $T = 500$ vs number of agents $n \in \{20, 50, 100, 200, 500, 1000\}$ under the (1,0) Taleb default. The empirical curve scales as $1/n^{0.95\pm0.05}$, matching the theoretical $1/n_{\text{eff}}$ prediction from Proposition 8.

A.3 Proof of Theorem 2 (Direction-DS)

[Full proof with the first-order expansions, the pivotal counterexample construction, and the asymptotic-pivotal-probability argument.]

A.4 Proof of Theorem 3 (BNE existence)

[Glicksberg fixed-point argument; uniqueness via best-response monotonicity in the symmetric case.]

A.5 Proof of Proposition 4 (early-time pool growth)

[Weighted Condorcet jury theorem under bounded weights, plus first-order expansion in ρ.]

A.6 Proof of Theorem 5 (reputation concentration)

[Mean-field drift derivation; martingale decomposition; Benaïm asymptotic-pseudo-trajectory argument; replicator-ODE convergence to vertex equilibrium.]

A.7 Proof of Lemma 6 (stake inheritance)

[Two-channel decomposition: signal-acceptance dominance + higher Kelly commitment.]

A.8 Proof of Proposition 8 (Taleb-corner drift)

[Per-period drift calculation under $(\zeta_+, \zeta_-) = (1, 0)$; Jensen inequality on $1/|Y_t^-|$.]

A.9 Proof of Lemma 9 (Gini sensitivity)

[Lorenz curve perturbation; HLP majorization formal version.]

A.10 Proof of Lemma 10 (EWMA filter)

[Linear systems theory: DC gain, transient bound.]

A.11 Proof of Theorem 12 (coalition extraction bound)

[Combining Lemmas 9, 10, 11 with Jensen.]

Appendix B — CRRA robustness

Under CRRA utility $u(x) = x^{1-\gamma}/(1-\gamma)$ with risk aversion $\gamma \neq 1$, the Kelly fraction generalizes to $c_i^* = (2q-1)x_i/\gamma$. Qualitative results: - T1' (direction-DS): holds. - T2' (BNE existence): holds for $\gamma > 0$. - T7 (impossibility): holds for any CRRA $\gamma > 0$, by the same Athreya-Karlin argument. - Proposition 8 (Taleb rate): the drift coefficient scales as $1/\gamma$ in the leading order; smaller risk aversion accelerates concentration.

Appendix C — Simulation Code and Reproducibility

The simulation code is provided in [supplementary URL or arXiv repository]. Key files: - `sim_CG1.py` — full mechanism simulation - `figures.py` — figure generation - `parameter_sweep.py` — sensitivity analysis

Random seeds for reproducibility: 42 for all main figures; seeds 100–149 for statistical confidence intervals.

C.1 Convergence diagnostics

[How we validate convergence to steady-state.]

C.2 Adversarial coalition simulation

[Details of the lockstep-voting adversarial coalition implementation used in §6.4.]

C.3 Head-to-head benchmarks

[Implementation details for the QV, futarchy, and LSSR-Stake comparison mechanisms.]

C.4 Strategic-anticipation regime

[Multi-period-ahead Kelly under strategic anticipation; Result 6 in §6.7.]

Appendix D — Alternative parameterizations

D.1 $\eta < 1$ (skill-premium split between commitment-weighted and flat-per-voter)

For $\eta \in (0, 1)$, the C1 payout becomes:

$$\Delta x_i = \mathbb{1}[i \in Y_t] \left[\eta \frac{c_i^t}{C_Y^t} + (1-\eta) \frac{1}{|Y_t|} \right] (1-\zeta_+) \Delta X_t + \zeta_+ \pi_i \Delta X_t.$$

Empirically: $\eta < 1$ slightly slows concentration (the flat portion is wealth-independent) but at the cost of weaker Kelly-commitment incentives. Recommended default keeps $\eta = 1$.

D.2 $\beta < 1$ (concave voting power)

$w_i^t = c_i^t \cdot r_i^\beta(t)$ or $w_i^t = (c_i^t)^\beta r_i(t)$ — concave voting power à la QV. Slows w-share concentration but does not directly affect c-share. Combined with c-share, the Lorenz cap $w_{\max}$ is a more effective anti-concentration overlay.

D.3 Lorenz cap $w_{\max}$

Cap individual voting weight at $w_{\max}$ as a fraction of $\sum_j w_j$. Slows reputation amplification under heterogeneous competence. Cost: reduces sharpness of the high-competence agent's voice on close calls.

D.4 Decay τ

Per-period pull of each x_i toward $\bar{x}$. Cost in per-period growth: approximately τ. Benefit: tighter long-horizon Gini bound.

These extensions provide additional knobs for deployment-specific tuning; we leave their joint optimization to future work.